Identifiability Before Interpretation: Conditional Octonion Transport, a No-Selection Theorem, and Bracketing Tests for a Seven-Component Candidate

Ivan Borisovich Kurpishev
Independent Researcher, Moscow, Russian Federation
me@kurpishev.ru

R-CP2, version 1.0 — 3 September 2026

Abstract

We separate an exact octonionic candidate from any physical identification of that candidate. For an oriented Euclidean seven-space, the compatible positive three-forms constitute the homogeneous space $\mathrm{SO}(7)/G_2$. We prove that the forgetful functor from metric G_2-spaces to oriented Euclidean seven-spaces has no natural section; hence dimension, metric, orientation, and an ordered list of seven labels cannot canonically select an octonion multiplication. Given an independently specified compatible positive three-form, we construct a choice-independent pullback product on $\mathbb{R} \oplus V$ and characterize the remaining G_2 gauge freedom. Applied to the seven labels of the $R \star R$ programme, this yields a conditional identification map $\Phi : V \to \mathrm{Im}(\mathbb{O})$, together with an explicit ledger of information still required for a physical map. The scalar unit e_0 is treated separately and is assigned an operational identity test, not a physical interpretation. Finally, the two bracketings of (e_1, e_2, e_4) are separated exactly by an e_7 readout, whereas every norm-only readout is blind to the difference. A preregistration-ready black-box protocol includes positive, null, identity, and calibration controls. Failure of identification or control gates returns Abstain; no physical claim is promoted by the mathematical results or by an emulator test.

Keywords. octonions; G_2-structure; identifiability; nonassociativity; associator; black-box protocol; falsifiability.
MSC 2020. 17A35; 53C10; 62F03; 81P99.

1 Purpose and status boundary

The octonions are the unique eight-dimensional real normed division algebra up to isomorphism, but uniqueness up to isomorphism does not identify a prescribed seven-component system with their imaginary subspace. The distinction is elementary and consequential. It separates three questions:

(Q1) Does a specified multiplication table define the standard real octonions?

(Q2) Which additional structure makes a map from seven source components to $\mathrm{Im}(\mathbb{O})$ mathematically checkable?

(Q3) What observation could distinguish the two bracketings of a triple in a physical or laboratory realization?

For the multiplication convention fixed in section 2, Q1 is answered affirmatively. The present paper addresses Q2 and Q3 without converting that mathematical fact into a physical claim. Its central result is an obstruction: the data currently attached to the seven source labels do not select a G_2-structure and therefore do not identify a map Φ.

This paper uses the following status rule throughout.

> *A theorem about a conditional octonion model changes only the status of that model. A physical status changes only after the source quantities, their units and readouts, the identification map, and an independently executed protocol have all passed their declared gates.*

If a required gate is absent or fails, the output is ABSTAIN. In particular, ABSTAIN is a determinate conclusion about identifiability; it is not evidence against octonions and not a substitute for a negative experimental result.

2 The fixed octonion candidate

Let $\mathbb{O} = \mathbb{R}e_0 \oplus \mathrm{Im}(\mathbb{O})$ with orthonormal basis $e_0, e_1, \ldots, e_7$. The element e_0 is the multiplicative unit, $e_i^2 = -e_0$, and the positively oriented Fano cycles are

$$123, \quad 145, \quad 176, \quad 246, \quad 257, \quad 347, \quad 365. \tag{1}$$

Thus $e_ie_j = e_k$ when (i, j, k) occurs cyclically in (1), and $e_je_i = -e_ie_j$ for distinct imaginary units. Conjugation and norm are

$$\overline{ae_0 + u} = ae_0 - u, \qquad N(ae_0 + u) = (ae_0 + u)\overline{(ae_0 + u)} = a^2 + \|u\|^2.$$

The norm is multiplicative. The associator is

$$K(x, y, z) = (xy)z - x(yz). \tag{2}$$

The algebra is alternative, so K is alternating. These conventions define a standard real octonion algebra; see [2, 3, 7].

Let $e^1, \ldots, e^7$ denote the dual basis. The multiplication convention is equivalently encoded by the positive three-form φ_0 defined by

$$\varphi_0(e_i, e_j, e_k) = 1 \quad \text{for the cyclic triples in (1)}, \tag{3}$$

extended by alternation. For $u, v, w \in \mathrm{Im}(\mathbb{O})$,

$$\langle u \times v, w\rangle = \varphi_0(u, v, w), \qquad uv = -\langle u, v\rangle e_0 + u \times v. \tag{4}$$

The stabilizer of φ_0 in SO(7) is the compact exceptional group $G_2 = \mathrm{Aut}(\mathbb{O})$ [4, 6].

The exact candidate therefore supplies a complete algebraic object. What it does not supply is a map from independently defined source quantities into that object.

3 Source labels and the type barrier

The source programme names seven components in the hierarchy

$$A \succ H_{px} \succ R_3 \succ R_2 \succ R_1 \succ \{i\hbar, iH\}. \tag{5}$$

The order relation in (5) is not, by itself, vector addition, scalar multiplication, an inner product, a three-form, or a multiplication table. Moreover, symbols such as $i\hbar$ and iH carry semantic and potentially dimensional information that cannot be erased merely by numbering the labels.

For bookkeeping only, introduce the free label space

$$V_{\mathcal{L}} = \operatorname{span}_{\mathbb{R}}\{s_A, s_{H_{px}}, s_{R_3}, s_{R_2}, s_{R_1}, s_{i\hbar}, s_{iH}\}. \tag{6}$$

Equation (6) is an editorial completion: it does not assert that the physical objects denoted by the labels can be added or rescaled.

To obtain a physical coordinate vector, a model must provide for each label ℓ:

$$X_\ell \supset \operatorname{Dom}(q_\ell) \xrightarrow{q_\ell} \mathbb{R}, \qquad \nu_\ell > 0, \qquad z_\ell(x) = \frac{q_\ell(x)}{\nu_\ell}, \tag{7}$$

where X_ℓ is a typed carrier, q_ℓ is an observable coordinate, and ν_ℓ is a declared reference scale with the same units as q_ℓ. Only then is

$$E(x) = \sum_{\ell\in\mathcal{L}} z_\ell(x) s_\ell \in V_{\mathcal{L}} \tag{8}$$

dimensionally meaningful. A change of any ν_ℓ changes the geometry unless the change is included in the model. No choice of units may be hidden inside Φ.

Definition 3.1 (Identification package)**.** An identification package for seven source components consists of:

(I1) typed carriers, domains, observables, and scales as in (7);

(I2) a positive-definite inner product g and orientation on the resulting real seven-space V;

(I3) a positive three-form χ compatible with g and the orientation;

(I4) an oriented isometry $\Phi : (V, g) \to (\operatorname{Im}(\mathbb{O}), \langle\cdot,\cdot\rangle)$ satisfying $\chi = \Phi^*\varphi_0$;

(I5) operational preparation, composition, and readout maps with declared uncertainty and failure domains.

Items (I1)–(I3) are source-side data. They cannot be inferred from the target octonion table without circularity.

4 No natural selection of the octonion structure

Let $\mathbf{Euc}_7^+$ be the groupoid of oriented Euclidean seven-spaces and orientation-preserving isometries. Let $\mathbf{G2Str}_7$ be the groupoid whose objects are triples (V, g, χ) with χ a positive three-form compatible with g and its orientation, and whose arrows preserve all three structures. Write

$$U : \mathbf{G2Str}_7 \longrightarrow \mathbf{Euc}_7^+, \qquad U(V, g, \chi) = (V, g)$$

for the forgetful functor.

Theorem 4.1 (No natural G_2 selection)**.** *The functor U has no natural section. Equivalently, there is no rule assigning to every oriented Euclidean seven-space (V, g) a compatible positive three-form $\chi_{V,g}$ such that every orientation-preserving isometry pulls the assigned form back to the assigned form.*

Proof. Suppose a natural section existed and evaluate it on standard oriented $\mathbb{R}^7$. Naturality would make the resulting three-form χ invariant under every element of $\mathrm{SO}(7)$. We show that an $\mathrm{SO}(7)$-invariant three-form must vanish.

Fix distinct indices i, j, k. Choose ℓ outside $\{i, j, k\}$, possible in dimension seven. The diagonal transformation that changes the signs of e_i and e_ℓ and fixes the other basis vectors lies in $\mathrm{SO}(7)$. It changes the sign of $\chi(e_i, e_j, e_k)$. Invariance therefore gives $\chi(e_i, e_j, e_k) = 0$. Every coordinate component vanishes, so $\chi = 0$. But a positive three-form is nonzero. This contradiction proves the claim. □

Proposition 4.2 (Size of the ambiguity)**.** *For fixed g and orientation on V, the compatible positive three-forms form a homogeneous space diffeomorphic to*

$$\mathrm{SO}(7)/G_2,$$

of dimension $21 - 14 = 7$*.*

Proof. Choose an oriented isometry $F : V \to \mathbb{R}^7$ and pull back the standard form φ_0. The group $\mathrm{SO}(7)$ acts transitively on compatible positive forms, and the stabilizer of φ_0 is G_2. The orbit–stabilizer description and dimension count follow. □

Proposition 4.3 (Explicit metric-preserving ambiguity)**.** *Let* $Q = \mathrm{diag}(-1, -1, 1, 1, 1, 1, 1) \in \mathrm{SO}(7)$ *and* $\varphi_1 = Q^*\varphi_0$*. The forms φ_0 and φ_1 induce the same metric and orientation, but are distinct. In particular,*

$$\varphi_0(e_1, e_4, e_5) = 1, \qquad \varphi_1(e_1, e_4, e_5) = -1.$$

The transformation Q is not an octonion automorphism.

Proof. The determinant of Q is $+1$, so the metric and orientation are preserved. The displayed component changes sign. Also $e_1 \times e_4 = e_5$, whereas

$$Q(e_1 \times e_4) = e_5 \quad \text{and} \quad (Qe_1) \times (Qe_4) = (-e_1) \times e_4 = -e_5.$$

Thus Q does not preserve the cross product and hence is not in G_2. □

Corollary 4.4 (Hierarchy-only nonidentifiability)**.** *An ordered hierarchy of seven labels, even after declaring its formal basis orthonormal and oriented, does not determine an octonion multiplication or a map to* $\mathrm{Im}(\mathbb{O})$*. The hierarchy* (5) *contains strictly less structure than the metric-and-orientation data in theorem 4.1.*

The obstruction is not a complaint about notation. It identifies the missing datum precisely: a compatible positive three-form, or equivalent cross product, must be supplied by an independent source-side rule or measurement.

5 Conditional construction of the map Φ

The negative theorem has a constructive complement.

Theorem 5.1 (Conditional octonion transport)**.** *Let (V, g) be an oriented Euclidean seven-space and let χ be a positive three-form compatible with g and the orientation. Define $\times_\chi$ by*

$$g(u \times_\chi v, w) = \chi(u, v, w). \tag{9}$$

On $\mathbb{R} \oplus V$ define

$$(a, u) \odot_\chi (b, v) = (ab - g(u, v),\ av + bu + u \times_\chi v). \tag{10}$$

Then $(\mathbb{R} \oplus V, \odot_\chi)$ is a unital normed algebra isomorphic to $\mathbb{O}$, with unit $(1, 0)$, conjugation $(a, u) \mapsto (a, -u)$, and norm $a^2 + g(u, u)$. Moreover, if an oriented isometry

$$\Phi : (V, g) \longrightarrow (\mathrm{Im}(\mathbb{O}), \langle \cdot, \cdot \rangle), \qquad \chi = \Phi^* \varphi_0, \tag{11}$$

is chosen, then

$$\widetilde{\Phi} : \mathbb{R} \oplus V \longrightarrow \mathbb{O}, \qquad \widetilde{\Phi}(a, u) = ae_0 + \Phi(u), \tag{12}$$

is an algebra isomorphism. The product (10) is independent of the admissible choice of Φ.

Proof. Compatibility means that an oriented isometry satisfying (11) exists. By (4) and (9),

$$\Phi(u \times_\chi v) = \Phi(u) \times \Phi(v).$$

Substituting this identity into the octonion product shows directly that $\widetilde{\Phi}((a, u) \odot_\chi (b, v)) = \widetilde{\Phi}(a, u)\, \widetilde{\Phi}(b, v)$. The claims about unit, conjugation, and norm follow by transport from $\mathbb{O}$.

If Φ_1 and Φ_2 are two admissible choices, then $A = \Phi_2 \Phi_1^{-1}$ preserves φ_0, hence $A \in G_2$. Its extension fixing e_0 is an octonion automorphism. Therefore both choices pull back the same product, namely (10). □

Corollary 5.2 (Gauge statement)**.** *For a fixed (V, g, χ), the admissible maps Φ form a right G_2-torsor. Observable statements invariant under target octonion automorphisms do not depend on a basis representative. Coordinate claims such as "the source component A is e_1" do depend on such a representative and require an additional convention or calibration.*

Remark 5.3 (What has and has not been constructed)*.* Theorem 5.1 gives a verifiable conditional map: a proposed triple (g, χ, Φ) can be checked by finitely many component equalities in a basis. It does not derive g, χ, or the encoders (7) from the labels in (5). For the present source record those inputs remain open; consequently the physical identification status is Abstain.

6 A finite verification contract for Φ

Let $(s_1, \ldots, s_7)$ be a declared source basis and write $G_{ij} = g(s_i, s_j)$, $C_{ijk} = \chi(s_i, s_j, s_k)$, and

$$\Phi(s_i) = \sum_{a=1}^{7} P_{ai} e_a.$$

A submission of G, C, and P passes the mathematical identification contract exactly when:

$$G = G^\mathsf{T} > 0, \qquad \det P > 0, \tag{13}$$

$$P^\mathsf{T} P = G, \tag{14}$$

$$C_{ijk} = \sum_{a,b,c=1}^{7} P_{ai} P_{bj} P_{ck} (\varphi_0)_{abc} \quad (1 \leq i, j, k \leq 7). \tag{15}$$

Condition (15) includes all ordered components; by alternation it is enough to check the 35 increasing triples. When exact arithmetic is not available, tolerances and the matrix norm must be fixed before evaluation.

For a proposed physical map, the following additional ledger is mandatory.

Field	Required record	Present disposition
Carrier	Type and domain X_ℓ for every source label	Open
Observable	Operational $q_\ell : X_\ell \to \mathbb{R}$	Open
Units/scale	Dimension and nonzero ν_ℓ	Open
Metric	Source-side construction of G	Open
Three-form	Source-side construction or measurement of C	Open
Map	Matrix P satisfying (13)–(15)	Conditional
Uncertainty	Error model and preregistered tolerances	Open
Replication	Independent implementation and data	Not run

The table is deliberately asymmetric. Target-side octonion identities cannot fill source-side cells.

7 The scalar unit e_0: mathematics and operational gate

The unit is not an eighth member of the seven-dimensional imaginary component list. Under theorem 5.1,

$$e_0 \longleftrightarrow (1,0) \in \mathbb{R} \oplus V, \qquad \Phi(V) = \mathrm{Im}(\mathbb{O}).$$

Mapping one of the seven labels to e_0 would reduce the imaginary image to at most six dimensions and violate the stated identification problem.

Algebra alone does not imply that e_0 is physical time, vacuum, an observer, or a cosmological background. The minimal operational interpretation is a *neutral composition preparation.* Let $\mathcal{P}$ be preparations, $\mu : \mathcal{P} \times \mathcal{P} \to \mathcal{P}$ a black-box binary gate, and D a declared output discrepancy. A preparation p_0 passes the unit gate at tolerance $(\varepsilon_0, \alpha_0)$ on a validation set $\mathcal{V}$ if

$$\Pr\left[\max_{p\in\mathcal{V}} \{D(\mu(p_0, p), p), D(\mu(p, p_0), p)\} \le \varepsilon_0\right] \ge 1 - \alpha_0. \tag{16}$$

Left and right tests are both required. Passing (16) supports the statement "p_0 implements the algebraic identity on $\mathcal{V}$ at the stated tolerance." It does not, without further theory and observation, give e_0 a physical ontology. Until such a preparation and calibration exist, the physical meaning of e_0 is Abstain.

8 An exact bracket-separating test

The Fano convention (1) gives

$$(e_1e_2)e_4 = e_3e_4 = e_7, \tag{17}$$

$$e_1(e_2e_4) = e_1e_6 = -e_7. \tag{18}$$

Consequently

$$K(e_1, e_2, e_4) = 2e_7. \tag{19}$$

The linear readout

$$S_7(q) = \langle q, e_7\rangle \tag{20}$$

returns $+1$ on the left branch and -1 on the right branch, with contrast 2. In contrast, multiplicativity of the octonion norm gives, for all x, y, z,

$$\|(xy)z\| = \|x(yz)\| = \|x\|\,\|y\|\,\|z\|. \tag{21}$$

Thus every protocol that observes only the final norm is structurally blind to bracketing. This is a no-go result for norm-only designs, not merely a warning about statistical power.

The triple (e_1, e_2, e_3) is a matched null-sector control:

$$(e_1e_2)e_3 = e_3e_3 = -e_0 = e_1e_1 = e_1(e_2e_3), \qquad K(e_1, e_2, e_3) = 0. \tag{22}$$

For a claimed source map Φ, set $u_j = \Phi^{-1}(e_j)$. Equations (17)–(19) pull back exactly, and the source readout is

$$S_\Phi(a, u) = g\big(u, \Phi^{-1}(e_7)\big), \qquad (a, u) \in \mathbb{R} \oplus V. \tag{23}$$

Without an accepted Φ, (23) is not an available physical observable; reporting the target coordinate S_7 would then be circular.

9 Preregistration-ready black-box protocol

The exact calculation becomes empirical only after an apparatus implements the preparations, binary gate, and readout. The machine-readable preregistration shipped with this paper is normative for parameter names; the following is its mathematical summary.

Protocol 9.1 (Hidden-branch e_1, e_2, e_4 test)**.** Before data acquisition:

(P1) Freeze the encoder and decoder, the preparations $p_0, p_1, \dots, p_7$, the binary gate μ, tolerances, exclusions, sample size, randomization seed commitment, and analysis code hash.

(P2) Calibrate preparation fidelity, both-sided identity behavior, and the binary products used in (17)–(18). Do not use either ternary composite from the primary trial to tune the device.

(P3) Within randomized blocks, conceal the branch label from the readout and analysis stages. Evaluate

$$L = \mu(\mu(p_1, p_2), p_4), \qquad R = \mu(p_1, \mu(p_2, p_4)).$$

(P4) Record the signed e_7 coordinate Y specified before acquisition. The primary estimand is

$$\theta = \mathbb{E}[Y \mid L] - \mathbb{E}[Y \mid R].$$

The ideal normalized octonion target is $\theta = 2$; the null of no bracket discrimination is $\theta = 0$.

(P5) Run the null triple $(1, 2, 3)$, identity insertions using p_0, a sham associative channel if available, and drift checks interleaved with the primary blocks.

The preregistered analysis reports effect estimate, confidence interval, raw and adjusted analyses, every exclusion, and all control outcomes. A two-sided confidence interval excluding 0 is necessary but not sufficient. It must also intersect the predeclared equivalence region about the calibrated octonion target, and every mandatory control gate must pass. Sample size is determined from a blinded pilot or an independently fixed noise bound and is not revised after unblinding.

The decision rule is:

Condition	Report
Any preparation, identity, drift, or readout control fails	ABSTAIN (invalid execution)
Controls pass, but interval includes 0	No detected bracket contrast at the declared resolution
Controls pass and primary/equivalence criteria pass on a programmable octonion emulator	Verified implementation of the candidate; no physical promotion
Controls pass on an independently motivated natural system with an accepted typed Φ	Physical-candidate evidence, still open pending independent replication
Source typing, normalization, or Φ is absent	ABSTAIN on correspondence regardless of signal size

The last two rows prevent a common category error. An emulator can test code, hardware, and analysis, but it cannot independently establish that the source system is octonionic.

10 Reproducibility contract

The release bundle contains an exact-arithmetic verifier. It represents basis units by signed coordinate vectors, generates the multiplication table from (1), and checks:

(V1) the complete 8×8 basis multiplication table;

(V2) $K(e_1, e_2, e_4) = 2e_7$ and $K(e_1, e_2, e_3) = 0$;

(V3) equality of the two branch norms and their signed e_7 contrast;

(V4) alternativity on the basis;

(V5) the count 168 of nonzero ordered imaginary-basis associator components, each equal to ± 2 in one imaginary coordinate, with zero scalar component;

(V6) the identity

$$\|K(x, y, z)\|^2 = 4\big(\det G(x, y, z) - \varphi_0(x, y, z)^2\big) \tag{24}$$

on a fixed suite of exact integer test vectors.

Here $G(x, y, z)$ is the 3×3 Gram matrix. The finite test suite for (24) is a regression check, not a proof; the identity itself is standard octonion geometry. The verifier emits a JSON report and exits with nonzero status on failure. Internal execution is labeled *internal*; it is not an independent replication.

11 Status ledger

The contribution and non-contribution of this paper can be read directly from the following ledger.

Item	Claim	Status after R-CP2
RCP2-M1	No natural metric/orientation-only selection of χ or Φ	Proved (mathematical)
RCP2-M2	Conditional transport from (V, g, χ) to $\mathbb{O}$	Proved (mathematical)
RCP2-X1	$(1, 2, 4)$ bracket contrast under S_7	Exact; internally verified
RCP2-X0	$(1, 2, 3)$ null-sector control	Exact; internally verified
RCP2-P1	Hidden-branch black-box design	Preregistration-ready; not run
Φ_{phys}	Source-to-Im($\mathbb{O}$) identification	Open / Abstain
e_0 ontology	Physical meaning beyond identity behavior	Open / Abstain
PO-1–PO-3	Physical $R \star R$ branch	Open; candidate closure only
PO-4–PO-8, PO-10	Remaining physical obligations	Open
F-1–F-14	Physical hypotheses	E0 / untested
EXT-RUN	Independent execution	Not run
Truth-layer promotion	Increase in physical status	0

Candidate-level closure means only that the stipulated algebra has the stated octonion properties. It cannot be cited as closure of the corresponding physical obligations.

12 Discussion

12.1 Why seven is necessary and insufficient

The imaginary octonions are seven-dimensional, so seven real coordinates are necessary for a linear isomorphism. But dimension matching supplies only a noncanonical element of $\mathrm{GL}(7, \mathbb{R})$. A metric reduces the freedom to $\mathrm{SO}(7)$ after orientation; an octonion cross product reduces it further to G_2. The remaining quotient $\mathrm{SO}(7)/G_2$ is exactly the ambiguity quantified in proposition 4.2. A numerical coincidence of cardinality therefore cannot perform the identification.

12.2 What would close the map obligation

There are two noncircular routes. A constructive route derives (g, χ) from independently defined source operations and proves positivity and compatibility. An empirical route estimates a source cross product or three-form from calibrated binary interactions, then validates it out of sample. In both cases the typed encoder (7) precedes the comparison with φ_0. Fitting a map after observing the bracket target without a held-out test does not identify the theory.

12.3 Relation to nonassociative physics

Nonassociative structures occur in mathematically controlled models of magnetic translations, flux backgrounds, and generalized quantum mechanics [9, 5, 8]. Those examples do not license transferring physical conclusions to a different source system. They instead illustrate the standard demanded here: the carrier, product, observables, and empirical or geometric origin must be specified together.

13 Conclusion

The main result is a controlled obstruction followed by a conditional construction. No canonical octonion identification follows from an oriented Euclidean seven-space, still less from seven ordered labels. Once a compatible positive three-form is supplied independently, the map Φ and the pulled-back octonion product are finite-checkable and unique up to G_2 gauge. The algebraic unit e_0 has a clear role, while any stronger physical meaning requires its own identity calibration and theory. The (e_1, e_2, e_4) test distinguishes the two bracketings by a signed coordinate with exact contrast 2; norm-only protocols cannot do so.

Accordingly, R-CP2 advances the mathematical candidate and the design of a falsifiable test, while returning Abstain on the physical correspondence. That firewall is part of the result.

Data and code availability

The release bundle contains the exact verifier, its machine-readable report, the preregistration record, and source files for this manuscript. No experimental data are reported. Re-execution of the verifier is internal unless performed by an independent party from the frozen bundle.

Declaration on computational and drafting assistance

OpenAI ChatGPT (Codex) was used for editorial drafting, proof-structure review, English-language assistance, and preparation of reproducibility code. The human author must review every statement, proof, reference, and software output before submission and retains full responsibility for the work. No AI system is listed as an author.

References

[1] H. Albuquerque and S. Majid, *Quasialgebra structure of the octonions*, J. Algebra **220** (1999), 188–224, arXiv:math/9802116.

[2] J. C. Baez, *The octonions*, Bull. Amer. Math. Soc. (N.S.) **39** (2002), 145–205, doi:10.1090/S0273-0979-01-00934-X, arXiv:math/0105155.

[3] R. B. Brown and A. Gray, *Vector cross products*, Comment. Math. Helv. **42** (1967), 222–236, doi:10.1007/BF02564418.

[4] R. L. Bryant, *Metrics with exceptional holonomy*, Ann. of Math. (2) **126** (1987), 525–576, doi:10.2307/1971360.

[5] S. Bunk, L. Müller, and R. J. Szabo, *Geometry and 2-Hilbert space for nonassociative magnetic translations*, Lett. Math. Phys. **109** (2019), 1827–1866, doi:10.1007/s11005-019-01160-4, arXiv:1804.08953.

[6] R. Harvey and H. B. Lawson, Jr., *Calibrated geometries*, Acta Math. **148** (1982), 47–157, doi:10.1007/BF02392726.

[7] R. D. Schafer, *An Introduction to Nonassociative Algebras*, Academic Press, New York, 1966.

[8] P. Schupp and R. J. Szabo, *An algebraic formulation of nonassociative quantum mechanics*, 2024, arXiv:2311.03647.

[9] R. J. Szabo, *An introduction to nonassociative physics*, PoS CORFU2016 (2017), 088, doi:10.22323/1.292.0088, arXiv:1709.10080.

A Project-specific identification record

The source hierarchy in (5) is recorded in the author document *Kurpishev PN2 author article*, version 0.2. The frozen first-volume source explicitly treats the seven-level interpretation as model structure and lists the carriers, transition maps, invariants, domains, distinguishability criterion, and empirical channel as prerequisites for a theorem. The passport *PN2 RstarR passport*, version 2.0-R, fixes the octonion convention used here, closes its algebraic checks only on the candidate branch, and designates the identification map, the physical meaning of e_0, and a bracket-discrimination protocol as the R-CP2 obligations.

These internal records establish provenance and scope, not external evidence. The present paper is self-contained with respect to its mathematical claims.

B Minimum independent replication report

An independent report should state:

(R1) the immutable bundle checksum and software environment;

(R2) whether the verifier was inspected or merely executed;

(R3) all deviations from the preregistration before unblinding;

(R4) raw trial-level outputs, branch assignments after reveal, and control outcomes;

(R5) the matrix P, metric G, tensor C, units, scales, and tolerances if a physical Φ is asserted;

(R6) separate conclusions for mathematical implementation, apparatus behavior, and source-system correspondence.

Missing items trigger Abstain for the corresponding layer and do not erase valid conclusions at lower layers.