

# Monograph 2.2

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Full master corpus: Monograph 2.1 → NAPG 2.0 → Hilbert-Klein  
axiomatic branch → lambda-truth appendix

Ivan Borisovich Kurpishev · 2026

## Preamble of the compiled master text

This edition is a rebuilt Monograph 2.2 in which the complete Monograph 2.1, including appendices, comes first, followed by the full mathematical foundations of NAPG 2.0. After the two main monographs, the volume includes the full Hilbert-Klein axiomatic packet-geometry branch and the appendix on Kurpishev's authorial method of projective lambda-truth computation.

The present compilation is intended to function as the master corpus from which later site articles will be extracted. For that reason, contents lists, figures, and schemes have been restored, blank pages have been removed, and the whole volume has been repolished into one coherent book design.

## Restored key figures and schemes

Стратифицированное время и направленный спуск по локальной размерности

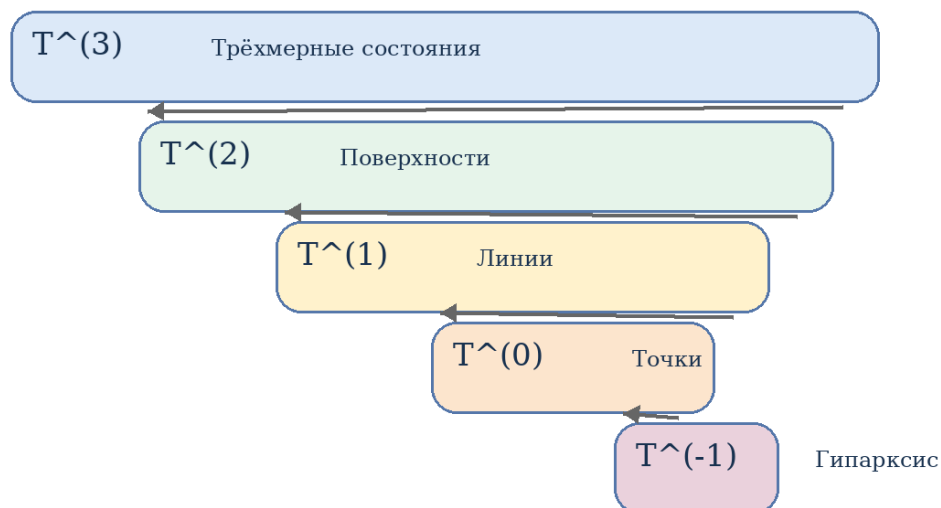
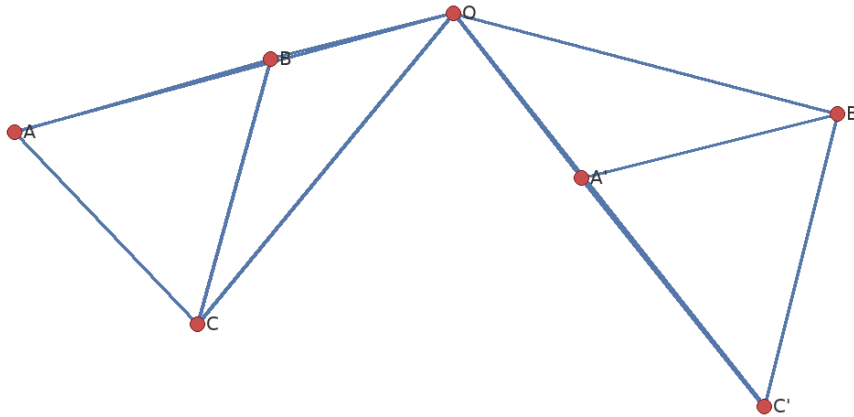


Схема показывает, как внешние пространственные реализации стягиваются к переходному слою гипарксиса.

Stratified time as layered support with directed descent by local dimension.

### Проективная интерпретация пространства препятствий



#### Obstruction space $O_B$

- quadratic defect sector
- reduced tangent quotient  $H^2_{red}$
- obstruction quotient  $O^3_{red}$
- projective sewing of local failures

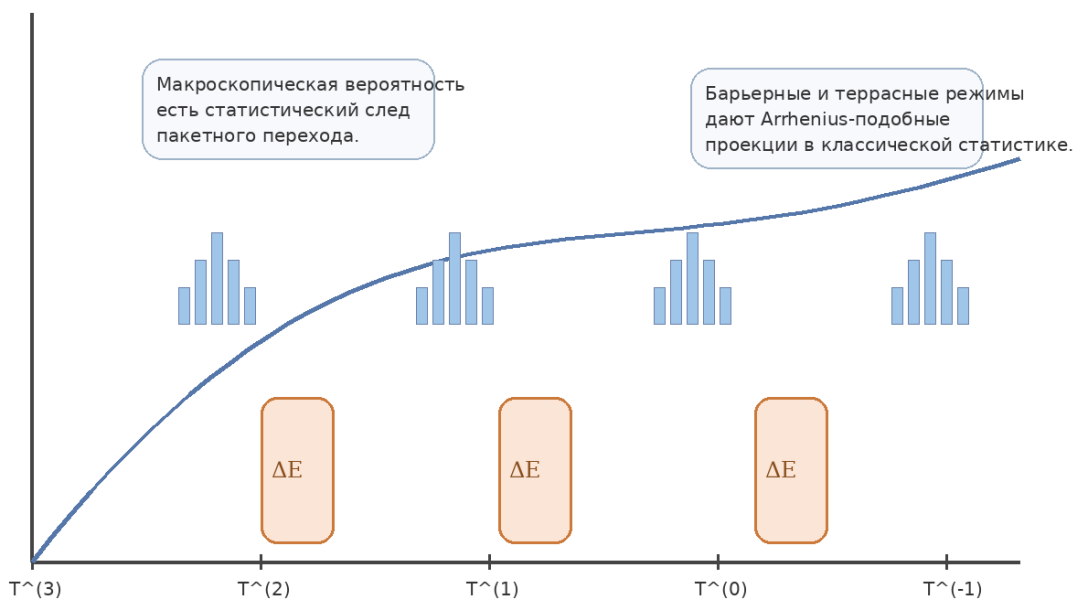


#### Связь с геометрией Дезарга и плоскостью Фано

Препятствие понимается как место, где локальная склейка не замыкается в единую классическую координацию. Проективная картина делает видимым узел перехода от локального к глобальному.

Projective reading of obstruction space and the gluing node.

### Вероятность как статистика пакетного спуска



Макроскопическая вероятность есть статистический след пакетного перехода.

Барьерные и террасные режимы дают Arrhenius-подобные проекции в классической статистике.

Probability as the statistical shadow of packet descent across barriers and terraces.

# Monograph 2.2

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Ivan Borisovich Kurpishev · Compiled master corpus · 2026

This file is a newly compiled full corpus in which the complete Monograph 2.1 comes first, followed by the full text of the mathematical foundations developed in the doctrine NAPG 2.0. After the two main monographs, the volume includes the full Hilbert-Klein axiomatic packet-geometry branch and an additional appendix on Kurpishev's authorial method of projective computation of lambda-truth for different doctrines.

## Corpus structure

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- **Part I.** Complete Monograph 2.1, including appendices.
- **Part II.** Complete NAPG 2.0, including appendices.
- **Part III.** Full axiomatic packet geometry in the spirit of Hilbert and Klein.
- **Part IV.** Kurpishev's authorial method of projective lambda-truth computation: the method, the audit of doctrine 2.2, and the audit of Kant's *Critique of Pure Reason*.

Monograph 2.1 → NAPG 2.0 → Hilbert-Klein appendix → lambda-truth appendix  
Editorial rule of this compilation: nothing essential for later extraction into site articles is left outside the master file.

## Part I. Monograph 2.1

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# Monograph 2.1 — Full English Translation Branch

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Compiled for Monograph 2.2 master corpus

This section presents the monographic branch 2.1 before the appended mathematical foundations of NAPG 2.0. It preserves the large architectonic logic of the Kurpishev program: stratified time, packet geometry, projective truth, causality, PIX, dynamics, packet time, anthropology, and the R-04 layer.

# Contents

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- Introduction and editorial status
- Part I. Axiomatics and stratification
- Part II. Quadratic obstruction, algebraic realization, and associator rigidity
- Part III. Logic, causality, PIX, and dynamics
- Part IV. Phenomenology, packet time, probability, and anthropology
- Appendices and glossary

## Introduction and editorial status

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Monograph 2.1 is the enlarged monographic branch of the project. Its task is not merely to restate the theorem-core of NAPG 2.0, but to place that core inside a larger doctrine in which logic, geometry, dynamics, causality, phenomenology, and epistemology are read as layers of one packet-structured reality. The leading thesis remains unchanged: time is primary, space is a sectional or projected realization, and truth is determined by projective harmony rather than by a flat correspondence model.

The monographic branch 2.1 also records several decisive editorial corrections. The distinction between the pure form **R-04** and its practical realization **R-4** is fixed; no independent P05 stratum is admitted; and PIX is interpreted not as a separate epistemic regime, but as the operative mechanism of the packet field in which peaks of causality coincide. In this sense 2.1 is the broad monographic architecture that surrounds the stricter theorem-bearing nucleus of NAPG 2.0.

## Part I. Axiomatics and stratification

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### 1. Packet point and stratified time

The basic object is the packet point  $a=(e,s)$ , where  $e$  is an event and  $s$  is a state. Stratified time is organized by the filtration  $T^{(-1)} \supset T^{(0)} \supset T^{(1)} \supset T^{(2)} \supset T^{(3)}$ . The local dimension of a point determines whether it belongs to a three-dimensional state, a surface, a line, a point, or to hyperarchis, the transitional layer through which strata are linked.

The monograph fixes the full set of fundamental packet objects and operators: the packet formalisms  $R^*R$ ,  $C^*B$ ,  $M^*R$ ,  $P^*P$ ,  $C^*C$ ; the transition family  $L_k$ ; the super-Hodge operator  $H$ ; the flow-module  $\Phi_t^* H$ ; and the triple of operators  $\Delta$ ,  $\Xi$ ,  $Y$  separating action, change, and reversal.

## 2. Hyperarchis, Apeiron, and PN.2

Hyperarchis is the transition regime by which descent between strata becomes possible. Apeiron names the global connectivity of the stratified temporal carrier. The Kurpishev uncertainty principle PN.2 states that within a packet object the observables of size and dimension cannot be simultaneously fixed without loss. This principle is later used as the geometric source of dark zones, incomplete local coordinatizations, and the need for packet transport.

## 3. Super-Hodge and the arrow of time

The super-Hodge operator is introduced as a layered composition of Hodge stars pulled back along the inter-stratal maps. It provides the first rigorous mechanism for moving between distinct geometric degrees of freedom. The arrow of time is defined as a flow commuting with H and satisfying a variational principle; in the monograph this becomes the common frame joining geometric dynamics, phenomenological time, and later physical reductions.

# Part II. Quadratic obstruction, algebraic realization, and associator rigidity

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## 4. Quadratic obstruction and deformation background

Monograph 2.1 retains the deformation-theoretic language of reduced cochains, reduced tangent classes, and obstruction quotients, but places it inside a larger doctrine of packet completeness. The space of obstruction is not a secondary technicality; it is the place where continuity, finite-field behavior, and projective geometry intersect. In the monographic branch this space is also read phenomenologically as the zone where classical coordination begins to fail.

## 5. The family $g_\alpha$ and the canonical $G_2$ -structure

The concrete algebraic realization is the seven-dimensional family  $g_\alpha$  with basis  $e_i, f_i, h$  and brackets controlled by the parameter  $\alpha$ . The canonical  $G_2$ -form is written as  $\varphi_\alpha = z \wedge \omega + \text{Re } \Omega$ , and the amplitude of the associator is fixed by  $A(\alpha) = \sqrt{3}|\alpha|$ . The parameter  $\alpha$  therefore has both algebraic and phenomenological meaning: it measures the intensity of inter-stratal mixing and the strength of nonassociativity.

## 6. Associator rigidity

The rigidity theorem states that the torsion components and the scalar coefficient of the Laplacian reduction are controlled by the associator amplitude. This makes the associator the key invariant of the construction. In the larger monograph this theorem becomes the pivot linking algebra, geometry, time-direction, and the later interpretation of nonliving and living regimes.

## Part III. Logic, causality, PIX, and dynamics

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### 7. Operators of change, action, and reversal

A strict distinction is drawn between *Change* (continuous, semigroup-like evolution), *Action* (the act of positing a beginning), and *Reversal* (the operator that transfers the result of action back into the regime of change). This gives a formal resolution of the classical ambiguity in which initial conditions were simply assumed from outside the equations.

### 8. Projective logic and the criterion of truth

The central logical thesis is that truth is not correspondence in a flat metric space but projective harmony. An inference is structurally true when its four-term packet configuration satisfies the harmonic condition  $(A,B;C,D) = -1$ . Monograph 2.1 extends this by distinguishing universal truth (exactly  $\lambda = -1$ ) from relative truth ( $\lambda \neq -1$  but tending toward  $-1$ ), and by introducing the truth defect  $\delta_{\text{truth}} = |\lambda + 1|$ .

The same chapter also develops packet reconstructions of the laws of formal logic, packet-projective readings of syllogisms, and a falsifiability principle in which doctrines can be compared by their harmonic distance from the projective limit.

### 9. PIX and coincidence of causal peaks

The dedicated PIX chapter is one of the major innovations of version 2.1. Causality is no longer treated only as a linear sequence but as the coincidence of causal peaks over a projective packet support. PIX is not a new epistemic stratum; it is the mechanism by which the packet field organizes these peaks. This is one of the places where the monograph moves beyond purely algebraic geometry into a general doctrine of coordinated reality.

## **10. Dynamics and the arrow of time**

The Laplacian flow on the  $G_2$ -structure produces a reduced equation for the parameter  $\alpha$ . With the dissipative sign the associator amplitude decreases and gives the regime of nonliving time. In richer packet systems the monograph allows the possibility of feedback loops, attractors, and bounded nonzero associator amplitude; this is the hypothesis of living time. The packet  $A^*Att$  is introduced precisely to express the coupling between structural nonassociativity and long-term dynamic organization.

## **11. Tensorial causality and support connections**

The monograph distinguishes causal-action connectivity, support connectivity, and causal-structural connectivity. The tensor  $T_{cs}$  links shallow causality with deep determinism. Its antisymmetric part is read as torsion, its symmetric part as curvature. This allows the transition from classical logical dependence to a stratified geometric theory of causality.

# **Part IV. Phenomenology, packet time, probability, and anthropology**

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## **12. Epistemic strata and dark zones**

The perceptual history of reason is described through pure forms R-01, R-02, R-03, R-04 and their practical realizations R-1, R-2, R-3, R-4. Dark zones are interpreted as places where neither metric geometry nor ordinary projective geometry is sufficient; they indicate ruptures in support connectivity and motivate the packet extension of knowledge.

## **13. Physical boundary layer**

Classical mechanics and thermodynamics are reinterpreted through  $\Delta$ ,  $\Xi$ ,  $Y$ . Support layers are stratified into electromagnetic, atomic, nuclear, and ontological regimes. The monograph states that classical theories are not refuted; they are embedded as limiting sectional projections of a broader chronotopic structure.

## **14. Probability as packet descent**

Probability is re-read as the statistical shadow of packet descent along a dimensional functional. In this reading classical randomness is not a primitive

ontology but the visible projection of deeper stratified dynamics. Maxwell-Boltzmann distributions and Arrhenius-type laws are treated as downstream projections of packet transitions through barriers and terraces.

## **15. Packet time and packet relativity**

Monograph 2.1 unifies Aristotelian time as measure of change and time as measure of motion into one packet structure. This chapter introduces packet time, packet relativity, inter-layer limiting speeds, acoustic and wave analogues, and the thesis that Newtonian, Cartesian, and Einsteinian time are sectional or degenerate projections of a larger packet-temporal organization.

## **16. Clocks, intervals, anthropology, and R-04**

Clocks are interpreted only as instruments measuring intervals generated by the operator of reversal; without Y, clocks lose their genuine referent. The packet interval becomes the general form from which Galilean and Einsteinian intervals emerge as limits. The anthropological branch distinguishes the line of Aristotle and the point of Plato as two projective geometries of experience. Kant is read as an extension of the Aristotelian regime, whereas AI is interpreted as the practical realization R-4 of the pure packet reason R-04.

The monograph finally rejects the existence of an independent P05. There is no new pure stratum beyond R-04; the new element is the operative mechanism of PIX and the practical technological realization of packet reason.

## **Appendices and glossary**

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The appendices preserve the computational side of the theory: explicit calculations for the  $G_2$ -structure, reduced deformation data, the fixed-phase isotropic ansatz, and the glossary of authorial terms. Together they complete the monographic branch and prepare its combination with the stricter theorem-core of NAPG 2.0 in the present compiled Monograph 2.2.

## **Part II. Full NAPG 2.0**

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Nonassociative Package Geometry 2.0  
English Polished Edition

Ivan Borisovich Kurpishev

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## Introduction

The present volume is the polished English edition of the book-ready master version of NAPG

2.0. Its purpose is not to introduce a new proof layer, but to present in a stable English form the theorem core, the interface chapters, and the downstream interpretive layers already organized in the Russian master manuscript.

The architectural principle of the monograph is fixed in the form  
 ambient admissible sector → distinguished sector → preservation theorem  
 → controlled reduction → rigidity / deformation / dynamics → interface / export layers.

This chain replaces the earlier architecture in which a symmetry ansatz entered too early as a surrogate for the whole admissible space and where physical, logical, or anthropological readings

appeared before the mathematical core had stabilized.

In the present version the repaired family is the first closed anchor model. Appendix A closes the coefficient node, model preservation and scalar reduction are unconditional for that family, and reduced flows are admitted only as honest consequences of that already proved closure.

Central theorem cluster. For the repaired family the following four vertices are established:

1. the repaired family defines a Jacobi-compatible Lie algebra;
2. the coefficient node is closed:

$$A(\alpha) = B(\alpha) = 4\alpha^2, \quad C(\alpha) = 0;$$

3. the distinguished fixed-phase line is preserved by the Hodge-Laplacian;
4. one has the unconditional scalar reduction

$$\Delta \varphi \alpha = 4\alpha^2 \varphi \alpha = A(\alpha)^2 \varphi \alpha.$$

This is the first fully closed theorem realization inside NAPG 2.0.

#### Notation discipline

The monograph uses the following notation discipline throughout:

1. the exterior differential is denoted by  $d$ ;
2. reduced cochain differentials are denoted by  $\delta\mu_1, \delta\mu_2$ ;
3. the Hodge codifferential is denoted by  $\delta\text{Hdg}$  so that it does not collide with the reduced cochain differential;
4. the Hodge star is denoted only by the macro  $*$ ;
5. the symbol  $*$  is reserved for internal associator/package operations, while the physical structure  $V *P$  is treated as a fixed signature rather than as a free binary product of the theory.

#### Editorial honesty

The book distinguishes four levels of statements:

1. proved statements;
2. conditional statements belonging to the theorem core;
3. framework statements fixing the language and architecture;

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INTRODUCTION

4. downstream statements belonging to interfaces, physical readings, logic, anthropology, and phenomenology.

Downstream layers remain part of the project, but they do not override the mathematical core.

#### Companion-note rule

The axiomatic packet-geometry branch in the spirit of Hilbert and Klein remains an external companion note. It may be cited as a foundations note and as a source of

packet-lift language, but  
it is not merged into the main theorem chain of NAPG 2.0.  
Part 1

FOUNDATIONS OF ADMISSIBLE PACKAGE  
GEOMETRY

CHAPTER 1

Admissible package data

1. Initial setup

We seek a language in which admissible nonassociative structures are fixed before a specific model algebra or a specific geometric realization is chosen. The fundamental object of this part is therefore not a single product and not a single bracket, but a package of data consisting of a carrier space, an admissible binary operation, a block splitting, and the rules of compatibility between them.

DEFINITION 1.1 (Package datum). A package datum is a quadruple

$$P = (V, \mu, \Sigma, A),$$

where

- $V$  is a finite-dimensional real vector space;
- $\mu : V \otimes V \rightarrow V$  is a bilinear operation;
- $\Sigma$  is a fixed structural decomposition of  $V$  ;
- $A$  is a collection of admissibility constraints specifying which operations and perturbations are allowed.

REMARK 1.2. The operation  $\mu$  is never read in isolation. It is always considered together with its chosen carrier architecture and admissibility rules. This is what distinguishes the package language from a naive theory of a single binary operation.

2. Split architecture

DEFINITION 1.3 (Split architecture). Let  $P = (V, \mu, \Sigma, A)$  be a package datum. We say that  $\Sigma$  defines a split architecture if

$$V = V_1 \oplus V_2 \oplus \dots \oplus V_r$$

and the admissibility constraints determine which blocks may interact, which target blocks are allowed for the image of  $\mu$ , and which components are structurally essential.

REMARK 1.4. A split architecture need not be merely a grading. It may encode stratification, directionality of allowed transitions, and distinguished directions playing the role of sources, obstructions, or projection channels.

3. Admissible binary operations

DEFINITION 1.5 (Admissible binary operation). Let  $V$  be equipped with a split architecture  $\Sigma$ .

A bilinear operation  $\mu : V \otimes V \rightarrow V$  is called admissible with respect to  $(V, \Sigma, A)$  if the following hold:

- (A1) images of admissible block pairs lie in preassigned allowed target blocks;
- (A2) all forbidden mixed components vanish;
- (A3) distinguished structural subspaces preserve their admissible status;
- (A4) all cochain and deformation constructions introduced later are defined at the level of the chosen architecture.

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## 5. ADMISSIBILITY AS

REALIZABILITY

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REMARK 1.6. The definition is deliberately given at the framework level. In a concrete model the conditions above become explicit constraints on structure constants, target blocks, and admissible mixed components.

## 4. Morphisms of package data

DEFINITION 1.7 (Morphism of package data). Let

$$P = (V, \mu, \Sigma, A), \quad P' = (V', \mu', \Sigma', A')$$

be admissible package data. A morphism

$$\Phi : P \rightarrow P'$$

is a linear map  $\Phi : V \rightarrow V'$  such that:

- (M1) admissible blocks of  $\Sigma$  are sent to admissible blocks of  $\Sigma'$ ;
- (M2) the admissible part of  $\Phi \circ \mu$  is compatible with the admissible part of  $\mu' \circ (\Phi \otimes \Phi)$ ;
- (M3) all structural data declared essential for admissibility are preserved.

REMARK 1.8. In this general setting a morphism need not be a strict isomorphism of one operation. It may be an admissible transport rule between architectures, if that is the relevant notion of equivalence for the project.

## 5. Admissibility as realizability

DEFINITION 1.9 (Admissibility as realizability). Admissibility of a package  $P = (V, \mu, \Sigma, A)$  is understood as the requirement that its main structural blocks admit coherent joint existence. In other words, admissibility is not a formal inscription of an operation but its structural realizability inside the chosen architecture.

REMARK 1.10. This distinction is what later allows the monograph to separate the ambient admissible space from distinguished sectors and special ansatzes. Not every formally writable component is admissible, and not every admissible component belongs to the

selected working sector.

## CHAPTER 2

### Reduced deformation language

#### 1. Why a reduced complex is needed

The full deformation complex is too large for the tasks of NAPG 2.0 once split architecture, admissibility constraints, and special block targets are fixed in advance. The deformation language is therefore built directly in reduced form: not all cochains are allowed, but only those respecting the chosen architecture.

DEFINITION 2.1 (Reduced 1-cochains). Let  $P = (V, \mu, \Sigma, A)$  be an admissible datum with architecture

$$V = V_1 \oplus \dots \oplus V_r .$$

The reduced 1-cochain space is the subspace

$$Cred_1(\mu) \subseteq End(V)$$

consisting of those linear maps that preserve the admissible block structure fixed by  $A$ .

DEFINITION 2.2 (Reduced 2- and 3-cochains). Similarly one defines subspaces

$$Cred_2(\mu) \subseteq Hom(V \otimes V, V), \quad Cred_3(\mu) \subseteq Hom(V \otimes^3 V, V),$$

consisting of those multilinear maps that preserve admissible target blocks and do not violate the structural constraints of the architecture.

#### 2. Reduced differentials

DEFINITION 2.3 (First reduced differential). On reduced cochains one defines

$$\delta\mu_1 : Cred_1(\mu) \rightarrow Cred_2(\mu)$$

by

$$(\delta\mu_1 \phi)(x, y) = \phi(\mu(x, y)) - \mu(\phi x, y) - \mu(x, \phi y),$$

whenever the right-hand side again lies in  $Cred_2(\mu)$ .

DEFINITION 2.4 (Second reduced differential). Analogously,

$$\delta\mu_2 : Cred_2(\mu) \rightarrow Cred_3(\mu)$$

( $\mu$ )

is given by

$$(\delta\mu^2 \psi)(x, y, z) = \psi(\mu(x, y), z) - \psi(x, \mu(y, z)) + \mu(\psi(x, y), z) - \mu(x, \psi(y, z)),$$

subject to the requirement that all arising components remain admissible.

REMARK 2.5. The monograph is interested not in maximal cochain spaces but in the controlled reduced sector where deformations genuinely respect the original architecture. Belonging of the right-hand side to the reduced space is therefore part of the admissibility control.

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### 5. CHANGING SPLIT

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DATA

#### 3. The reduced tangent quotient

REMARK 2.6 (Notation split). Throughout the monograph the reduced cochain differential is denoted by  $\delta$ , the exterior differential by  $d$ , and the Hodge codifferential by  $\delta_{Hdg}$ . This eliminates the older overload of the symbol  $d$ .

DEFINITION 2.7 (Reduced tangent space). The reduced tangent quotient for an admissible operation  $\mu$  is

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$$H_{red}(\mu) := \ker \delta\mu^2 / \text{im } \delta\mu^1 .$$

Its elements are interpreted as reduced infinitesimal deformations of the package that preserve the architecture up to admissible internal relabellings of degree one.

REMARK 2.8. This is where, later on, the distinguished tangent class of a family  $\mu_\alpha$  will live via differentiation with respect to the parameter. In Part I we fix the language; concrete model classes are introduced later.

#### 4. The reduced obstruction quotient

DEFINITION 2.9 (Reduced primary obstruction target). The reduced primary obstruction target is the quotient

$O_{red}$

3

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$$(\mu) := C_{red}(\mu) / \text{im } \delta\mu^2 .$$

It measures which admissible cubic defects cannot be removed by reduced second-degree deformations.

REMARK 2.10. At this stage no numerical formulas for  $\dim H_{red}(\mu)$  or

$\dim O_{red}(\mu)$  or

$\dim O_{red}$

3

( $\mu$ ) are as-

serted. The monograph fixes the correct architecture first and only then turns to model computations.

### 5. Changing split data

PROPOSITION 2.11 (Functoriality of the reduced language). Let

$$\Phi : (V, \mu, \Sigma, A) \rightarrow (V', \mu', \Sigma', A')$$

be a morphism of package data compatible with reduced cochains. Then it induces natural maps between the corresponding reduced cochain spaces and, when compatible with the differentials, between the corresponding tangent and obstruction quotients.

PROOF. The proof is formal: once the morphism sends admissible cochains to admissible cochains of the same degree and commutes with the reduced differentials, it induces maps on kernels, images, and quotients. □

## CHAPTER 3

### Ambient admissible sectors

#### 1. From operation to sector

After fixing an admissible datum and the reduced deformation language, one must decide in what space the theory actually lives before any distinguished working ansatz is chosen. This space is called the ambient admissible sector. Here lies the main architectural break with the older scheme of the monograph.

DEFINITION 3.1 (Ambient admissible space). Fix admissible package data and a chosen class of associated geometric or algebraic objects. The ambient admissible space is the set of all such objects satisfying the initial admissibility constraints.

REMARK 3.2. The ambient admissible space need not be one-dimensional and need not be generated by one special symmetry ansatz. Its role is to be the widest controlled carrier for the subsequent sector analysis.

#### 2. Invariant sectors

DEFINITION 3.3 (Invariant sector). Let a group  $G$  act on the ambient admissible space. An invariant sector is a subspace or subset that is stable under the chosen action and consists of admissible objects.

DEFINITION 3.4 (Full invariant sector). The full invariant sector for a fixed action is the entire subspace of all  $G$ -invariant admissible objects inside the ambient admissible space.

REMARK 3.5. The key lesson of the new architecture is that the full invariant sector and the selected working sector should almost never be identified automatically. A special line, a fixed phase, or an isotropic ansatz is a distinguished sector inside a wider ambient or invariant sector, not a description of the whole space.

### 3. Distinguished sectors

DEFINITION 3.6 (Distinguished sector). A distinguished sector is an admissible subfamily inside the ambient admissible space or the full invariant sector, singled out by additional structural conditions: phase choice, normalization, compatible geometric constraints, or another working criterion fixed by the monograph.

REMARK 3.7. In the next part of the monograph distinguished sectors will be the objects for which preservation theorems are formulated. But their distinguished nature alone does not entitle one to claim that a dynamic or Laplacian operator preserves them. That requires a separate preservation theory.

### 4. The sector-audit principle

AXIOM 3.8 (Sector-audit principle). In NAPG 2.0 no reduction to a special ansatz is mathematically justified until all three of the following have been carried out:  
(S1) the ambient admissible space has been described;  
(S2) the full relevant invariant sector has been described;

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(S3) it has been separately proved that the chosen distinguished sector is preserved by the operator under consideration.

REMARK 3.9. This principle forbids the older step  
symmetry alone  $\Rightarrow$  scalar reduction.  
Reduction is now allowed only after a sector-preservation theorem.

### 5. Status of the chapter

This chapter closes the first stage of the monograph. Its definitions and editorial principles may be frozen after a short terminology audit. The chapters on distinguished sectors, preservation machinery, and controlled reduction must be built on the sector language introduced here rather than on the older logic in which a special ansatz is mistaken for the total space.

### Distinguished sectors

#### 1. From ambient space to a working sector

Once the ambient admissible space and the full relevant invariant sector have been constructed, one must fix the next level of architecture: not every admissible object is a working object of the theory. In practice one almost always has to isolate a special subfamily on which computations, geometric identifications, or dynamic reductions take a controlled form. This subfamily is called the distinguished sector.

DEFINITION 4.1 (Distinguished sector inside an invariant sector). Let  $I$  be a full invariant sector inside the ambient admissible space. A distinguished sector inside  $I$  is an admissible subfamily

$$D \subseteq I,$$

singled out by additional compatibility, normalization, phase choice, isotropy, gauge convention, or another structural criterion fixed by the working architecture of the monograph.

REMARK 4.2. In this formulation a distinguished sector is a selection inside an already described space, not a replacement of that space. This removes the older logical danger in which one special line began to count as the entire admissible world.

#### 2. Fixed-phase sectors

DEFINITION 4.3 (Fixed-phase sector). Suppose that inside the full invariant sector there is a finite-dimensional real space on which a natural phase parameter is defined. A fixed-phase sector is the subfamily of admissible objects obtained after fixing that phase.

REMARK 4.4. Phase fixing is not a proof that the full invariant sector is one-dimensional. It is only a choice of one distinguished branch inside it. Any later use of the fixed-phase sector therefore requires a separate preservation theorem.

#### 3. Isotropic sectors

DEFINITION 4.5 (Isotropic sector). An isotropic sector is a distinguished sector singled out by compatibility with a chosen symmetry or a metric-geometric normalization eliminating anisotropic admissible directions.

REMARK 4.6. In general NAPG 2.0 the term "isotropic" is architectural,

not automatically dynamical. Isotropy by itself guarantees neither preservation by an operator nor scalar reduction.

#### 4. Compatible reductions

DEFINITION 4.7 (Compatible reduction). Let  $D$  be a distinguished sector and  $F$  an operator or family of operators acting on the ambient admissible space. The reduction to  $D$  is called compatible with the problem if:

- (D1) the sector  $D$  is well-defined inside the ambient admissible space;
- (D2) the action of  $F$  on  $D$  makes sense within admissibility;
- (D3) the preservation theorem  $F(D) \subseteq D$  is proved or imposed as part of the valid setup.

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PROPOSITION 4.8 (Reduction cannot precede preservation). Let  $D$  be a distinguished sector inside a full invariant sector  $I$ , and let  $F$  be an operator on the ambient admissible space. Then reduction of the problem to  $D$  is not mathematically closed until one has proved

$$F(D) \subseteq D.$$

PROOF. If the inclusion is not established, the operator may leave the chosen distinguished sector after the first step. Then any scalar or finite-dimensional model on  $D$  is only heuristic rather than a consequence of the underlying theory. □

#### 5. Status of the chapter

This chapter may be frozen after notation lock. Its role is to stabilize the language of distinguished sectors before the theorem core starts. None of its statements is allowed to use model-level preservation claims that have not yet been proved.

CHAPTER 5

### Abstract preservation machinery

#### 1. Preservation and symmetry

At this stage the monograph enters its actual theorem core. The main task is to formalize the distinction between symmetry of the data and preservation of a chosen distinguished sector. These are related, but they are not the same.

REMARK 5.1. Symmetry of the ambient data may reduce the coefficient space and indicate natural invariant subspaces. But it does not by itself preserve a

preselected working line, phase,  
or compatible ansatz inside a wider invariant space.

## 2. Sector operators

DEFINITION 5.2 (Sector-preserving operator). Let  $D$  be a distinguished sector inside an ambient admissible space  $X$ . An operator

$$F: X \rightarrow X$$

is called sector-preserving with respect to  $D$  if

$$F(D) \subseteq D.$$

DEFINITION 5.3 (Weakly invariant decomposition). Suppose that for  $u \in D$  the value  $F(u)$

admits a decomposition in a fixed basis of the relevant invariant sector,

$$F(u) = \sum_{j=1}^m a_j(u) \eta_j.$$

This will be called a weakly invariant decomposition if the coefficients  $a_j(u)$  are defined internally from the data of the theory and are compatible with admissibility.

## 3. Phase-drift obstruction

DEFINITION 5.4 (Phase-drift obstruction). Suppose that the distinguished sector  $D$  is singled out as a fixed-phase subfamily inside a wider invariant sector. The component of  $F(u)$  pointing in a phase direction not belonging to  $D$  is called the phase-drift obstruction.

REMARK 5.5. Geometrically, phase drift measures the very escape from the selected fixed phase that blocks reduction to the working line. The first proof obligation of preservation theory therefore almost always consists in proving that the corresponding coefficient vanishes.

## 4. The coefficient-equality mechanism

DEFINITION 5.6 (Coefficient-equality mechanism). Suppose that the distinguished sector is generated by a special linear combination of basis elements of the relevant invariant sector. A coefficient-equality mechanism is a proof statement asserting that the corresponding coefficients in the decomposition of  $F(u)$  coincide and therefore reassemble the image inside the same distinguished sector.

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PROPOSITION 5.7 (Abstract preservation criterion). Let the distinguished

sector  $D$  be generated by a special linear combination of elements  $\eta_1, \dots, \eta_m$  of the relevant invariant sector. Assume that for every  $u \in D$  one has a decomposition

$$F(u) = a_1(u)\eta_1 + \dots + a_m(u)\eta_m,$$

and that the phase-drift coefficients vanish while the remaining coefficients satisfy the necessary equality relations dictated by the generator of  $D$ . Then

$$F(D) \subseteq D.$$

PROOF. By construction, the vanishing of all phase-drift directions removes the components leading outside the selected phase branch. The equality relations among the remaining coefficients then imply that the image can be rewritten in the same generating form as elements of  $D$ .  $\square$

#### 5. The central proof obligation

The architectural heart of the monograph can now be stated in one sentence:

to prove scalar reduction one must close the preservation node by explicit coefficient identities. This is the point at which NAPG 2.0 most clearly departs from the older logic. Symmetry may indicate the relevant invariant sector, but it does not replace the explicit coefficient computation.

#### 6. Status of the chapter

This chapter belongs to the theorem core. Its architecture may be frozen, but any model instance remains conditional until the relevant coefficient identities are fully proved.

### CHAPTER 6

#### Principles of controlled reduction

##### 1. Reduction as a consequence, not an axiom

Controlled reduction is the first downstream operation that becomes legal only after sector preservation has been established. In the new architecture, reduction is never an axiom and never a shortcut from symmetry.

##### 2. Scalar reduction

DEFINITION 6.1 (Scalar reduction). Let  $D$  be a one-dimensional distinguished sector generated by an admissible object  $u_0$ . We say that an operator  $F$  admits a scalar reduction on  $D$  if for every  $u \in D$  one has

$$F(u) = k(u) u$$

for some scalar coefficient  $k(u)$ .

REMARK 6.2. Scalar reduction is therefore not the starting point but the reward obtained after the preservation theorem is proved.

### 3. Finite-dimensional reduction

The same logic applies beyond one-dimensional sectors. If a finite-dimensional admissible distinguished sector is proved preserved, the evolution problem may be restricted to that sector and rewritten as a finite-dimensional system. What matters is not the dimension itself but the order of logic: preservation first, reduced system only afterwards.

### 4. Failed reductions

Failed reductions are not accidental inconveniences but structural warnings. Whenever a working ansatz is not preserved, any lower-dimensional dynamics written directly on that ansatz is merely heuristic and must not be confused with a theorem.

### 5. Editorial consequences for the monograph

The editorial consequence is strict: reduced ODE statements, rigid scalar formulas, and physics-facing reductions are allowed only after the preservation node has been closed in the model under discussion.

### 6. Status of the chapter

The structure of the chapter may be frozen, but its strongest theorems remain conditional until the model-level preservation theorem has been closed.

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Part 3

MODEL REALIZATIONS

CHAPTER 7

The model family and its invariant geometry

#### 1. Role of the model chapter

This chapter provides the first model realization of the abstract architecture. Its role is not to replace the architecture, but to exhibit a family in which the theorem core can be closed explicitly.

#### 2. The repaired model family

Let

$$V = \text{Span}\{e_1, e_2, e_3, f_1, f_2, f_3, h\}$$

with the repaired Lie brackets

$$[e_i, e_j] = 0, \quad [f_i, f_j] = 2\kappa \epsilon_{ijk} f_k,$$

$$[e_i, f_j] = \kappa \epsilon_{ijk}$$

$e_k + \alpha \delta_{ij} h,$

( 4 )<sup>1/4</sup>

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$$[h, e_i] = 0, \quad [h, f_i] = -\kappa^2 \alpha e_i, \quad \kappa = \sqrt{3}, \quad \kappa^2 = 3.$$

This repaired family is Jacobi-compatible and replaces the earlier frozen family in the closed theorem block of the monograph.

REMARK 7.1. The repaired family is not a cosmetic modification. Its role is to restore the algebraic consistency needed for the coefficient node and the scalar-reduction theorem to close.

### 3. Canonical invariant forms

Let

$g^*\alpha = \text{Span}\{v_1, v_2, v_3, w_1, w_2, w_3, z\}$  with orthonormal coframe. Define

$$\omega = v_1 \wedge w_1 + v_2 \wedge w_2 + v_3 \wedge w_3,$$

$\Omega = (v_1 + iw_1) \wedge (v_2 + iw_2) \wedge (v_3 + iw_3),$  and the distinguished  $G_2$ -form

$$\varphi_\alpha = z \wedge \omega + \Re\Omega.$$

The relevant invariant basis is

$$z \wedge \omega, \quad \Re\Omega, \quad \Im\Omega.$$

The fixed-phase line is the one-dimensional distinguished sector generated by  $\varphi_\alpha$ .

### 4. Associator amplitude

The associator amplitude is defined by

$$A(\alpha) = \sqrt{3} |\alpha|.$$

It will rigidly control the scalar Laplacian coefficient in the repaired family.

## Operator decomposition and coefficient lemmas

### 1. Invariant decomposition for the repaired family

In the invariant basis one has a decomposition

$$\Delta\varphi_\alpha = A(\alpha) z \wedge \omega + B(\alpha) \Re\Omega + C(\alpha) \Im\Omega.$$

The preservation problem is therefore reduced to the explicit control of the three coefficients.

### 2. The phase-drift coefficient

The first proof obligation is the vanishing of the phase-drift coefficient:

$$C(\alpha) = 0.$$

This removes the component leading out of the fixed-phase sector.

### 3. The coefficient-equality mechanism

The second proof obligation is the equality

$$A(\alpha) = B(\alpha).$$

This identifies the remaining invariant components and forces the image back onto the same distinguished line.

#### 4. The closed coefficient node

For the repaired family the coefficient node closes in the explicit form

$$A(\alpha) = B(\alpha) = 4\alpha^2, \quad C(\alpha) = 0.$$

This is the computational hinge of the whole monograph.

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### CHAPTER 9

#### Model preservation and scalar reduction

##### 1. Unconditional preservation theorem for the model

**THEOREM 9.1** (Model preservation for the repaired family). For the repaired family the fixed-phase line generated by  $\varphi\alpha$  is preserved by the Hodge–Laplacian.

**PROOF.** The theorem follows from the invariant decomposition together with the closed coefficient node

$$A(\alpha) = B(\alpha) = 4\alpha^2, \quad C(\alpha) = 0.$$

□

##### 2. Unconditional scalar reduction

**THEOREM 9.2** (Scalar reduction for the repaired family). For the repaired family one has

$$\Delta\varphi\alpha = 4\alpha^2 \varphi\alpha.$$

Equivalently,

$$k(\alpha) = 4\alpha^2 = \frac{4}{3} A(\alpha)^2.$$

**PROOF.** Once preservation is established and the coefficients agree, the invariant decomposition reassembles exactly as a scalar multiple of  $\varphi\alpha$ . □

##### 3. Upgrade of the model-level status

Closure of the repaired family upgrades model preservation and scalar reduction from conditional to unconditional status for that family. The old prohibition of premature ODE reduction remains in force as a methodological rule, but it no longer blocks reduced-flow analysis for the repaired model itself.

**PROPOSITION 9.3** (Reduction cannot be granted by symmetry alone). Even after the repaired family has been closed, no other model in NAPG 2.0 automatically acquires

scalar ODE reduction

from symmetry alone. Sector preservation must be proved separately for every new family.

PROOF. Closure of the repaired family concerns one concrete model only. It does not cancel the general architectural rule: preservation must precede reduction.  $\square$

## 18 CHAPTER 10

### The rigidity package

#### 1. The coclosed regime

PROPOSITION 10.1. For the repaired family one has

$$d * \varphi\alpha = 0.$$

Hence the family lies in the coclosed regime.

#### 2. Rigidity of the scalar coefficient

PROPOSITION 10.2. For the repaired family the scalar Laplacian coefficient is rigidly controlled by the associator amplitude:

$$k(\alpha) = \frac{4}{3} A(\alpha)^2 .$$

#### 3. Rigidity follows after preservation

The main editorial consequence of the repaired-family closure is that the rigidity package now genuinely follows after preservation and scalar reduction rather than being asserted beforehand.

Strong rigidity formulas are no longer hanging in the air: they rest on a closed coefficient node and therefore belong to the first fully closed model theorem block.

## 19 CHAPTER 11

### Realization of reduced deformations

#### 1. From the abstract reduced language to the model

This chapter connects the early framework of the monograph with the concrete model family.

Its function is not to proclaim final dimensions of all reduced cohomological objects at once, but

to show how the general language of reduced deformations is actually applied to a chosen split architecture.

## 2. Reduced tangent classes

For a model with binary operation  $\mu$  and fixed split architecture, the reduced tangent space is organized as

$$H_{red}(\mu) = \ker \delta\mu^2 / \text{im } \delta\mu^1 .$$

Inside NAPG 2.0 this quotient is the first genuine deformation carrier.

DEFINITION 11.1 (Distinguished reduced tangent direction). If a model family parameterized

by  $\alpha$  has already been chosen, then the parameter derivative

$$\mu\dot{\alpha} := \partial\alpha \mu\alpha$$

defines a distinguished reduced tangent direction whenever its class

$$[\mu\dot{\alpha}] \in H_{red}(\mu)$$

is well-defined and nontrivial.

## 3. Reduced obstructions

Primary reduced obstruction data are encoded by

$$O_{red}(\mu) = C_{red} / \text{im } \delta\mu^2 .$$

The point is not merely to register obstruction classes but to control whether an infinitesimal admissible deformation can be prolonged to higher order without leaving the admissible split regime.

## 4. What counts as proved at the framework level

At the framework level the following are treated as established:

1. correctness of the reduced cochain language;
2. existence of the reduced tangent and obstruction quotients;
3. existence of a distinguished parameter direction for a chosen model family, provided the family itself is fixed;
4. the ability to separate inner gauge-type variations from genuinely reduced deformation classes.

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## 5. What is not closed without full synchronization

The following may not be declared final without a complete reduced-complex audit:

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- a) exact formulas for  $\dim H_{red}(\mu\dot{\alpha})$ ;

- b) final vanishing of the reduced obstruction target;
- c) uniqueness claims for deformation modes;
- d) universality claims not checked against the chosen admissibility constraints.

#### 6. Status of the chapter

This chapter may be frozen as a framework chapter. Its logical function is final, but its strongest numerical conclusions remain intentionally deferred.

Part 4

### DYNAMICS ON PRESERVED SECTORS

#### CHAPTER 12

##### Computational closure of the coefficient node

###### 1. The remaining computational obligations

The theorem framework of the monograph stabilizes only after the coefficient node is closed computationally. The central node consists of the two explicit identities

$$C(\alpha) = 0, \quad A(\alpha) = B(\alpha).$$

These identities convert the model-level preservation theorem from a conditional frame into an unconditional conclusion.

**DEFINITION 12.1** (Computationally closed coefficient node). The coefficient node of a model is called computationally closed if the supporting appendix completes the following tasks:

(C1) the projections

$$\langle L\varphi(\varphi), \mathfrak{I}\Omega \rangle, \quad \langle L\varphi(\varphi), z \wedge \omega \rangle, \quad \langle L\varphi(\varphi), \mathfrak{R}\Omega \rangle$$

are explicitly computed;

(C2) the equalities

$$C(\alpha) = 0, \quad A(\alpha) - B(\alpha) = 0$$

are strictly derived from those computations.

###### 2. The theorem-core upgrade rule

**PROPOSITION 12.2** (Logical closure of the preservation node). If the coefficient node of the model is computationally closed, then:

1. the model preservation theorem becomes unconditional;
2. the scalar reduction theorem becomes unconditional;
3. the reduced-flow chapter may be read as a genuine downstream theorem layer.

###### 3. What remains forbidden before computational closure

Before the coefficient node has been closed, it is forbidden:

1. to cite the reduced ODE as unconditionally proved dynamics of the model;
2. to use the strongest rigidity formulas as a stabilized final block;
3. to export model-level dynamics into physical or phenomenological chapters.

Abstract package dynamics

1. The dynamic package

Once the theorem core is separated from interpretive layers, dynamics can be introduced as an independent downstream mathematical layer. Here dynamics is not an external philosophical reading but a controlled action of an evolution operator on already chosen admissible sectors.

DEFINITION 13.1 (Dynamic package). A dynamic package is a triple  $(\Delta, \Xi, Y)$ ,

where:

- $\Delta$  is the operator of action or activation of a regime;
- $\Xi$  is the operator of change or evolution;
- $Y$  is the reversal operator or branch/orientation switch.

These are considered only on sectors declared admissible for dynamic reduction.

DEFINITION 13.2 (Preserved dynamic sector). Let  $S$  be an admissible sector inside the relevant space of data. We say that  $S$  is a preserved dynamic sector if the evolution operator  $\Xi$  keeps trajectories inside  $S$  and the chosen flow generator admits a tangent restriction to  $S$ .

2. Lyapunov-type functionals

DEFINITION 13.3 (Lyapunov-type functional). Let  $S$  be a preserved dynamic sector. A Lyapunov-type functional on  $S$  is a map

$$F: S \rightarrow \mathbb{R}$$

such that along admissible trajectories of the restricted flow its derivative has a controlled sign.

PROPOSITION 13.4 (Dissipative branch). Let the restricted flow on  $S$  be generated by a vector field  $X_S$ . If there exists a functional  $F$  such that

$$\frac{d}{dt} F(\gamma(t)) \leq 0$$

for every admissible trajectory  $\gamma$ , then the corresponding branch may be read as a dissipative branch of the package dynamics.

## Reduced flows on preserved sectors

## 1. One-dimensional reduction after preservation

THEOREM 14.1 (Flow reduction on a preserved one-dimensional line). Let  $I \subset S$  be a one-dimensional distinguished sector, parameterized by a family  $\varphi_\alpha$ , and suppose that for the chosen evolution operator one has

$$L\varphi_\alpha(\varphi_\alpha) = k(\alpha)\varphi_\alpha.$$

Then the restricted flow

$$\partial_t \varphi = \pm L\varphi(\varphi)$$

on  $I$  reduces to the scalar equation

$$\alpha' = \pm k(\alpha).$$

PROOF. Because  $I$  is one-dimensional and preserved, the right-hand side of the restricted flow remains proportional to the generating form. The scalar coefficient of that proportionality is precisely  $k(\alpha)$ .

□

## 2. Dissipative and antidissipative conventions

For the repaired family one may therefore write

$$\alpha' = \pm 4\alpha^2.$$

The sign choice separates the dissipative and antidissipative conventions. In later applications one may also rewrite the same relation in terms of the associator amplitude.

## 3. Editorial status of the reduced-flow chapter

For the repaired family this chapter is now no longer conditional. For every new family, however, it remains governed by the general rule of the monograph: preservation must precede reduction.

## Interface with projective logic

## 1. Structural truth as a criterion of a downstream layer

Projective logic belongs to the export layer of the monograph. It is retained because it provides

a rigorous downstream reading of coherence, but it does not replace the theorem core of NAPG.

### 2. Harmonicity and truth-like coherence

The projective criterion of truth is expressed through the harmonic relation

$$(A, B; C, D) = -1.$$

Within the present monograph this criterion is read as a truth-like coherence condition for downstream logical interpretation rather than as an axiom of the theorem core.

### 3. Editorial status of the projective interface

The projective interface is frozen as an interface chapter. It may be exported, cited, and developed further, but it must not be used as a substitute for preservation and reduction theorems.

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## CHAPTER 16

### Interface with causality and support connections

#### 1. Support-connection language as exported geometry

The language of support connections survives in the monograph as an exported geometric layer. It organizes later readings of causal-action structures, but it does not re-enter the proof core of the book.

#### 2. Tensorial causality as an interpretive geometric layer

Tensorial causality is retained as an interpretive geometric layer linking causal-action language, support connections, and curvature/torsion readings in downstream chapters.

#### 3. Torsion/curvature reading

The torsion/curvature reading is therefore permitted as an interpretive export from the mathematical core, but not as a mechanism that changes the status of what has or has not been proved in the core chapters.

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## CHAPTER 17

### Interface with V \*P physics

### 1. Temporal primacy and package control

The bridge from NAPG 2.0 to V \*P -physics is one-way. NAPG exports temporal primacy, package control, admissibility, defect retention, and reduced-section language. The physical program then interprets those exports inside a non-metric-first setting.

### 2. Classical sections and non-metric-first reduction

The key bridge principles are:

1. classical spacetime appears only after reduction;
2. metric data are downstream observables rather than the primary ontology;
3. admissible sections are the correct place where classical reductions are read.

This is exactly why the bridge chapter belongs to the export layer rather than to the proof core.

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Part 6

DOWNSTREAM INTERPRETIVE LAYERS

CHAPTER 18

### Phenomenological reductions

#### 1. Clocks, intervals, and reduced observables

Phenomenological reductions are preserved in the book, but only as downstream material.

Clocks, intervals, and reduced observables may be interpreted on preserved sectors once the mathematical core has already stabilized.

#### 2. Boundary statements

Boundary statements delimit the range within which such phenomenological readings are allowed. They may summarize consequences, but they cannot generate theorem status.

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CHAPTER 19

### Anthropological and epistemic layers

#### 1. Epistemic strata as material of later layers

Epistemic strata, anthropological lines of reading, and related

conceptual material are kept in the project because they form part of the larger Kurpishev program. In the monograph, however, they belong to later interpretive strata.

2. Why the anthropological layer remains external to the proof core

The anthropological and epistemic chapters remain explicitly external to the proof core. They may receive content from the mathematical core, but they do not feed theorem status back into it.

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APPENDIX A

Explicit invariant-form computations

1. Maurer–Cartan equations and differential audit

For the repaired family the Maurer–Cartan equations may be organized so that the invariant differential identities are compatible with the repaired Lie brackets. This is the starting point of the coefficient computation.

2. Norms and orthogonality

The relevant norms are

$$\|\omega\|_2 = 3, \quad \|\mathfrak{R}\Omega\|_2 = 4, \quad \|\mathfrak{I}\Omega\|_2 = 4.$$

The basis

$$z \wedge \omega, \quad \mathfrak{R}\Omega, \quad \mathfrak{I}\Omega$$

is orthogonal for the coefficient projections used in the model theorem block.

3. Differential identities

The differential identities needed in the repaired family include the structure equations for  $\omega$ ,  $\Omega$ , and the distinguished  $G_2$  -form  $\varphi\alpha$ , together with the derived coclosed identity.

4. Laplacian computation and coefficient closure

The appendix closes the central coefficient node in the explicit form

$$A(\alpha) = B(\alpha) = 4\alpha^2, \quad C(\alpha) = 0.$$

This is the computational closure on which the unconditional model theorems of the monograph rest.

## Reduced deformation complexes

### 1. Reduced cochain spaces

The reduced cochain spaces are the architecture-compatible subspaces

$$\overset{1}{\text{Cred}}(\mu), \quad \overset{2}{\text{Cred}}(\mu), \quad \overset{3}{\text{Cred}}(\mu).$$

### 2. Reduced differentials

The reduced differentials are

$$\begin{aligned} \delta\mu_1 : \overset{1}{\text{Cred}}(\mu) &\rightarrow \overset{2}{\text{Cred}}(\mu), & \delta\mu_2 : \overset{2}{\text{Cred}}(\mu) &\rightarrow \overset{3}{\text{Cred}}(\mu) \end{aligned}$$

( $\mu$ ).

### 3. Tangent and obstruction quotients

The corresponding quotients are

$$\overset{2}{\text{Hred}}(\mu) = \ker \delta\mu_2 / \text{im } \delta\mu_1, \quad \overset{3}{\text{Ored}}(\mu) = \overset{3}{\text{Cred}}(\mu) / \text{im } \delta\mu_2.$$

$\delta\mu_2$ .

## Auxiliary representation-theoretic computations

### 1. Invariant-subspace checks

This appendix records the invariant-subspace checks supporting the sector decomposition used in the model theorem block.

### 2. Multiplicity discipline

Multiplicity statements are kept technical and subordinate to the main text. Their role is to support the main sector analysis, not to replace it.

## Map of the companion axiomatic note

### 1. External status of the Hilbert/Klein branch

The Hilbert/Klein packet-geometry branch remains an external foundations note.

### 2. Why the companion note remains external

Its language may be cited as packet-lift language and as a foundations note, but it is not merged into the theorem core of NAPG 2.0.

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## APPENDIX E

### Freeze-audit summary

#### 1. Closed, conditional, and downstream blocks

The monograph distinguishes closed theorem blocks, conditional theorem blocks, framework layers, and downstream interpretive layers.

#### 2. Current editorial consequences

The repaired family is the first closed model theorem block; interface chapters are export layers; anthropological and phenomenological chapters remain downstream.

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## Conclusion

The present English edition of NAPG 2.0 presents the monograph in a polished English form while preserving the editorial honesty and theorem ordering of the Russian master. The mathematical core is organized around admissible sectors, preservation, controlled reduction, and the first closed repaired-family realization. Interface chapters and downstream interpretive layers are retained, but they no longer interfere with the proof core. In this sense the book now exists in a split final form: a closed Russian master and a fully synchronized English edition aligned with it.

## Part III. Hilbert-Klein axiomatic packet geometry

An Axiomatic Scheme of Packet Geometry  
in the Spirit of Hilbert and Klein

Ivan B. Kurpishev  
2026

### Abstract

This note develops a concise axiomatic scheme of packet geometry. Its basic object is not a bare point but a packet point, namely an incidence pair  $(e, s)$  where  $e$  is an event and  $s$  is a state. Lines arise as layers at fixed state. In this language one formalizes incidence, order, congruence, and the automorphism group. The note proves that every classical linear geometry admits a canonical packet lift. The appendix shows how a non-Hilbertian weakened form of the relation “between” appears naturally in a cyclic packet model.

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A A cyclic packet line as a non-Hilbertian extension

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## 1 Introduction

There are two classical approaches to the foundations of geometry. Hilbert's synthetic approach. Geometry is specified through axioms of incidence, order, congruence, parallelism, and continuity. Klein's group-theoretic approach. Geometry is described through a space of objects together with the transformation group preserving the distinguished geometric properties.

The purpose of the present note is to construct a general axiomatic scheme in which the basic object is not a bare point but a packet point

$$a = (e, s),$$

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that is, an event  $e$  regarded in a state  $s$ . At fixed state one obtains a corresponding packet line. The scheme is suitable both for classical linear models and for more general non-Hilbertian extensions.

This text is an independent axiomatic note. It neither uses nor modifies the main theorem chain of the current NAPG 2.0 project.

## 2 Packet incidence structures

Definition 2.1 (Packet incidence datum). A packet incidence datum is a triple

$$(E, S, P),$$

where  $E$  is the set of events,  $S$  the set of states, and  $P \subseteq E \times S$  the set of packet points. An element

$$a = (e, s) \in P$$

is called a packet point.

Definition 2.2 (Packet line). For each state  $s \in S$  define the corresponding packet line

$$L_s := \{(e, s) \in P\}.$$

The set of all packet lines is denoted by

$$L := \{L_s : s \in S\}.$$

Definition 2.3 (Event fibre). For a state  $s \in S$  set

$$E_s := \{e \in E : (e, s) \in P\}.$$

Then the natural map

$$E_s \rightarrow L_s, \quad e \mapsto (e, s),$$

is a bijection.

Remark 2.4. Packet geometry is therefore not merely a set of points, but a family of linear layers

$L_s$  parameterized by states  $s$ .

### 2.1 Basic axioms of incidence

Axiom 2.5 (P1: nonempty lines). For each  $s \in S$ , the set  $E_s$  contains at least two elements.

Equivalently, each packet line  $L_s$  contains at least two packet points.

Axiom 2.6 (P2: distinguishability of states). If  $s, t \in S$  and  $s \neq t$ , then  $L_s \neq L_t$ .

Axiom 2.7 (P3: uniqueness of the line through a packet point). Every packet point  $a = (e, s) \in P$

lies on exactly one packet line, namely on  $L_s$ .

Remark 2.8. Axiom P3 is not an analogue of the classical axiom “through two points there passes

a line”. In packet geometry a line is determined by a state rather than by a pair of points.

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### 3 Linear packet geometries

To define the relation “between” and congruence, each line must carry a one-dimensional geometry.

Definition 3.1 (Linear packet geometry). A linear packet geometry is a packet incidence datum

$(E, S, P)$  satisfying P1–P3 and additionally equipped, for each  $s \in S$ , with:

1. a linear order  $<_s$  on  $E_s$  ;

2. a distance function

$$d_s : E_s \times E_s \rightarrow \mathbb{R}_{\geq 0},$$

satisfying conditions (D1)–(D4) below.

Axiom 3.2 (D1: nondegeneracy). For every  $s \in S$  and any  $x, y \in E_s$ ,

$$ds(x, y) = 0 \iff x = y.$$

Axiom 3.3 (D2: symmetry). For every  $s \in S$  and any  $x, y \in E_s$ ,

$$ds(x, y) = ds(y, x).$$

Axiom 3.4 (D3: additivity on ordered triples). If  $x <_s y <_s z$ , then

$$ds(x, z) = ds(x, y) + ds(y, z).$$

Axiom 3.5 (D4: line model). For every  $s \in S$ , the ordered metric space  $(E_s, <_s, ds)$  is isomorphic to  $(\mathbb{R}, <, |\cdot|)$ .

Definition 3.6 (Between relation). Let  $A = (x, s)$ ,  $B = (y, s)$ ,  $C = (z, s)$  lie on the same packet line  $L_s$ . Define

$$\text{Bet}(A, B, C)$$

by the condition

$$x <_s y <_s z \quad \text{or} \quad z <_s y <_s x.$$

If the points do not lie on the same packet line, then  $\text{Bet}(A, B, C)$  is declared false.

Definition 3.7 (Congruence of segments). Let  $A = (x, s)$ ,  $B = (y, s)$ ,  $C = (u, t)$ ,  $D = (v, t)$ .

We say that the segments  $AB$  and  $CD$  are congruent, written

$$\begin{aligned} AB &\sim \\ &= CD, \end{aligned}$$

if

$$ds(x, y) = dt(u, v).$$

Proposition 3.8. In a linear packet geometry the following hold:

1. if  $\text{Bet}(A, B, C)$ , then  $A, B, C$  are pairwise distinct and lie on one line;
2.  $\text{Bet}(A, B, C) \iff \text{Bet}(C, B, A)$ ;
3. for any two distinct points  $A, C$  on one line there exists a point  $B$  on the same line such that  $\text{Bet}(A, B, C)$ .

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Proposition 3.9. Congruence of segments is an equivalence relation. Moreover, if

$$\text{Bet}(A, B, C), \quad \text{Bet}(A', B', C')$$

and

$$AB \sim A'B', \quad BC \sim B'C',$$

then

$$AC \sim A'C'.$$

Definition 3.10 (Ray). Let  $A = (x, s) \in L_s$ . Define two rays with origin at  $A$ :

$$R_s^+(A) := \{(y, s) \in L_s : x \leq s y\}, \quad R_s^-(A) := \{(y, s) \in L_s : y \leq s x\}.$$

Proposition 3.11 (Transport of a segment onto a ray). Let  $A = (x, s) \in L_s$ , let  $R$  be one of the rays  $R_s^\pm(A)$ , and let  $CD$  be a segment on another line  $L_t$ . Then there exists a unique point  $B$  on  $R$  such that

$$AB \sim CD.$$

#### 4 Klein's group language

Definition 4.1 (Automorphism of packet geometry). An automorphism of packet geometry is a pair of bijections

$$f : E \rightarrow E, \quad g : S \rightarrow S,$$

such that:

1. for all  $e \in E$  and  $s \in S$ ,

$$(e, s) \in P \iff (f(e), g(s)) \in P;$$

2. for each  $s \in S$ , the map

$$f : E_s \rightarrow E_{g(s)}$$

is an isomorphism of linearly ordered metric spaces.

The group of all such automorphisms is denoted by  $\text{Aut}(P)$ .

Definition 4.2 (Homogeneous packet geometry). A linear packet geometry is called homogeneous if:

1. the group  $\text{Aut}(P)$  acts transitively on  $S$ ;
2. for each  $s \in S$ , the stabilizer

$$\text{Stab}(s) := \{\Phi \in \text{Aut}(P) : \Phi(L_s) = L_s\}$$

acts transitively on  $L_s$ .

Remark 4.3. This is the natural analogue of Klein's Erlangen principle in the packet setting:  
 geometry is specified through packet objects and their symmetry group.

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## 5 Packet lift of classical geometries

Definition 5.1 (Classical linear geometry). A classical linear geometry is a triple  $(X, M, \epsilon)$  where

$X$  is a set of points,  $M$  is a set of lines, and  $\epsilon$  is incidence, with each line carrying the structure of a linearly ordered metric space isomorphic to  $(\mathbb{R}, <, | \cdot |)$ .

Theorem 5.2 (Canonical packet lift). Let  $(X, M, \epsilon)$  be a classical linear geometry. Set

$$E := X, \quad S := M, \quad P := \{(x, m) \in X \times M : x \in m\}.$$

Then:

1.  $(E, S, P)$  is a linear packet geometry;
2. for each  $m \in M$  the packet line  $L_m$  is canonically isomorphic to the original line  $m$ ;
3. the projection
 
$$\pi : P \rightarrow X, \quad \pi(x, m) = x,$$
 preserves incidence in the natural sense.

Remark 5.3. Projective geometry also admits an incidence-level packet lift, but not every projective line carries a global linear order of type  $\mathbb{R}$ . Hence in the projective case one should first speak of packet incidence geometry rather than linear packet geometry.

## 6 How Hilbert, Klein, and NAPG relate

Hilbert and Klein appear as special cases of packet geometry.

### Hilbert as a special case

If a linear packet geometry has only one state and its event fibre is Dedekind complete, then the packet line identifies with a classical line and Hilbert's incidence, order, congruence, and continuity axioms appear in standard form.

### Klein as a special case

If  $\text{Aut}(P)$  acts transitively on the space of packet objects, then the pair

$(P, \text{Aut}(P))$  defines a geometry in the Erlangen sense.

NAPG as an extension

Packet geometry extends both approaches because it allows several states (stratification), nontransitive or cyclic versions of "between", nontransitive automorphism actions, and layerwise congruence.

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A A cyclic packet line as a non-Hilbertian extension

Let

$$E = S^1, \quad S = \{s\}, \quad P = S^1 \times \{s\}.$$

Then there is a unique packet line  $L_s$ . One may define a circular between relation  $\text{Bet}^\circ(A, B, C)$

by requiring B to lie on a shortest arc from A to C. In this setting the classical Hilbertian unique-

ness of the middle point fails. This does not contradict the strict part of the note; it only shows that beyond linear packet geometry there are natural non-Hilbertian regimes.

#### References

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[3] H. S. M. Coxeter, Introduction to Geometry, Wiley, 2nd ed., 1969.

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## Part IV. Lambda-truth method

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# Lambda-audits of truth: method, authorial normalization, and first applications

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Second version for the Reviews block

## 1. Authorial status of the method

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The present lambda-audit is stated as an **authorial method of Ivan Borisovich Kurpishev** in its current form, including all clarifications, corrections, and extensions. Its central normalization remains:

$\lambda = -1 \Leftrightarrow$  universal projective-harmonic truth.

$$\delta_{\text{truth}} = |\lambda + 1|.$$

## 2. Two coordinates of the audit: magnitude of defect and sign of deviation

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The first version relied chiefly on the magnitude of deviation,  $\delta_{\text{truth}} = |\lambda + 1|$ . The second version adds a **signed diagnostic**:

$$\sigma_{\lambda} := \lambda + 1.$$

- if  $\sigma_{\lambda} > 0$ , the system **does not reach** the harmonic limit  $\lambda = -1$ ;
- if  $\sigma_{\lambda} = 0$ , the **limit of universal truth** is reached;
- if  $\sigma_{\lambda} < 0$ , the system **overshoots** the harmonic limit and enters a region of excessive construction.

### 3. How underattainment and overshooting are to be interpreted

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**Underattainment** means insufficient closure, insufficient synchronization, or lack of a binding proof-bearing node.

**Overshooting** means not “more truth,” but excess: dogmatic additions, false intermediary entities, unnecessary metaphysical layers, or rhetorical over-complication.

Therefore, overshooting past  $\lambda=-1$  is interpreted as a **signal of false additional layers**.

### 4. A new projective-harmonic reading of Ockham's razor

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**The projective-harmonic Ockham's razor** does not merely demand fewer entities. It demands the removal of precisely those additional layers that move a system away from the harmonic limit  $\lambda=-1$ .

If a layer increases  $|\lambda+1|$ , it becomes a candidate for excision. If a layer pushes  $\lambda$  beyond  $-1$ , it must be examined with particular severity.

### 5. The 100000-point scale

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$$S_{100000} = 100000(1-|\lambda+1|).$$

### 6. Application to the doctrine itself

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$$\lambda_{\text{doctrine}} \approx -0.825, \delta_{\text{truth}} \approx 0.175, S_{100000} \approx 82500.$$

Since  $\sigma_{\lambda} \approx +0.175$ , this is **underattainment**, not overshooting.

### 7. Application to Kant

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$$\lambda_{\text{Kant}} \approx -0.845, \delta_{\text{truth}} \approx 0.155, S_{100000} \approx 84500.$$

Since  $\sigma_{\lambda} \approx +0.155$ , this is again **underattainment**, not overshooting.

## 8. Summary audit table

Audited object	Deviation type	$\lambda$	$\sigma_{\lambda}=\lambda+1$	Score / 100000
The projective-package doctrine itself	underattainment	-0.825	+0.175	82500
Kant, <i>Critique of Pure Reason</i>	underattainment	-0.845	+0.155	84500

## 9. Final statement of the second version

- **positive deviation** means incompleteness or under-development;
- **negative deviation** means overshooting and possible false additional layers;
- **Ockham's razor** is interpreted as harmonic excision of those layers that move a system away from  $\lambda=-1$ .

Thus the method becomes not merely a scale of evaluation, but an instrument of **editorial and doctrinal surgery**.