On the Potential Along a Tangent to an Equipotential Circle and the Geometric Mechanism of the "Uphill Flow" Illusion

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Abstract

We consider a simple model of a central field on the Euclidean plane in which the potential depends only on the distance to a fixed center. We prove that the restriction of such a potential to a tangent line of an equipotential circle attains a strict maximum at the tangency point. It follows that, along the tangent, the direction of increasing potential is always oriented toward the tangency point and not toward a more distant visible summit. We then separate this exact geometric fact from the visual illusion often described as "uphill flow." The illusion is explained by incomplete line-of-sight information: a hidden lower point of the true profile may be invisible to the observer, which leads to a false perception of the slope.

1 Introduction

Consider a central field with center at a point O, an equipotential circle S_{r_0}, a tangency point $O' \in S_{r_0}$, and the tangent line l at O'. Let A and B be two points on l, lying on opposite sides of O'.

Two questions arise naturally:

1) how does the potential behave along the tangent line l;

2) why, in real observation, can one still obtain the impression that a road or a flow goes "uphill," even though the mathematical model does not predict such behavior.

The first question is purely geometric. The second belongs to interpretation and perception: one must distinguish the true profile from the visible profile available to the observer.

2 Mathematical Model

Definition 2.1. Let $\mathbb{R}^2$ be the Euclidean plane with a fixed point O. Suppose a scalar function

$$\Phi(X) = f\big(|OX|\big)$$

is given, where $f : (0, +\infty) \to \mathbb{R}$ is strictly decreasing. Thus Φ depends only on the distance from X to the center O.

Assumption 2.2. For every $r > 0$, the circle

$$S_r = \{X \in \mathbb{R}^2 : \ |OX| = r\}$$

is equipotential, and

$$r_1 < r_2 \quad \Longrightarrow \quad \Phi|_{S_{r_1}} > \Phi|_{S_{r_2}}.$$

Equivalently, the value of Φ strictly decreases as the distance from O increases.

Definition 2.3. Fix a number $r_0 > 0$, the circle S_{r_0}, and a point $O' \in S_{r_0}$. Let l be the tangent line to S_{r_0} at O'. Choose points $A, B \in l$ on opposite sides of O'.

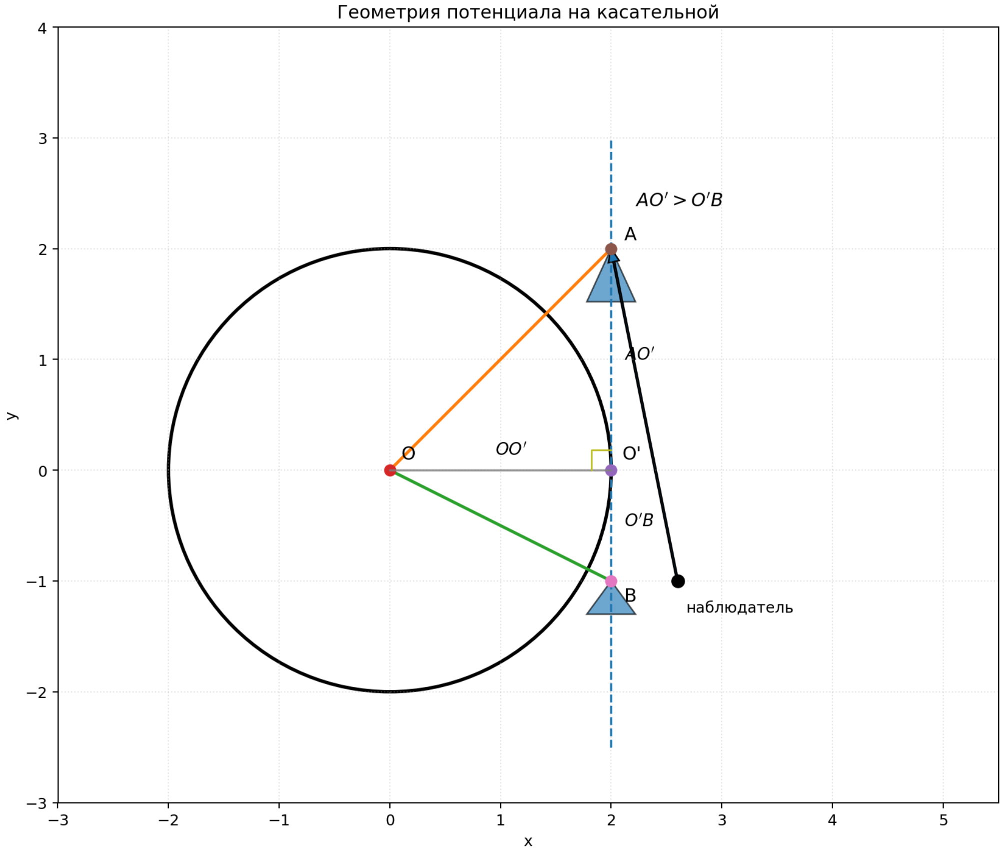

Figure 1: Geometry of the potential on the tangent. Among all points of the tangent line, the point O' is the closest one to the center O. Therefore the restricted potential $\Phi|_l$ attains its maximum at O'.

Lemma 2.4. *For every point $X \in l$ one has*

$$|OX|^2 = |OO'|^2 + |XO'|^2 = r_0^2 + |XO'|^2.$$

Consequently, if $X, Y \in l$ and

$$|XO'| < |YO'|,$$

then

$$|OX| < |OY|.$$

Proof. Since the radius OO' is perpendicular to the tangent line l at O', the triangle XOO' is right-angled at O'. By the Pythagorean theorem,

$$|OX|^2 = |OO'|^2 + |XO'|^2 = r_0^2 + |XO'|^2.$$

Hence $|OX|$ is a strictly increasing function of $|XO'|$. □

Theorem 2.5 (Potential on the Tangent)**.** *Let $X, Y \in l$. If*

$$|XO'| < |YO'|,$$

then

$$\Phi(X) > \Phi(Y).$$

In particular, the restricted potential $\Phi|_l$ attains a strict maximum at the tangency point O'.

Proof. By Lemma 2.4, the inequality $|XO'| < |YO'|$ implies $|OX| < |OY|$. Since $\Phi(X) = f(|OX|)$ and f is strictly decreasing, it follows that

$$\Phi(X) > \Phi(Y).$$

If $X = O'$, then $|XO'| = 0$, and therefore for every $Y \in l \setminus \{O'\}$,

$$\Phi(O') > \Phi(Y).$$

□

Corollary 2.6. *If the points A and B are chosen so that*

$$|AO'| > |BO'|,$$

then

$$|OA| > |OB|, \qquad \Phi(A) < \Phi(B) < \Phi(O').$$

Remark 2.7. Corollary 2.6 fixes the crucial point: if A is farther from the tangency point O' than B, then in a central decreasing field the potential at A is smaller than the potential at B. The tangent geometry by itself therefore does not yield a model in which a liquid should spontaneously flow from B toward the farther summit A.

3 Geometric Mechanism of the Illusion

Theorem 2.5 describes the true distribution of the potential. Human perception, however, does not operate with the entire profile; it uses only the visible part of that profile. If a lower point of the road is hidden from view, then the visible line may be interpreted incorrectly.

This situation is illustrated in Figure 2. The observer is located near B and sees the distant summit A, but does not see the hidden lower point near O'. In such a configuration, the comparison between the “visible height on one side” and the “visible height on the other side” becomes unreliable. The true profile contains a depression that is not visually controlled.

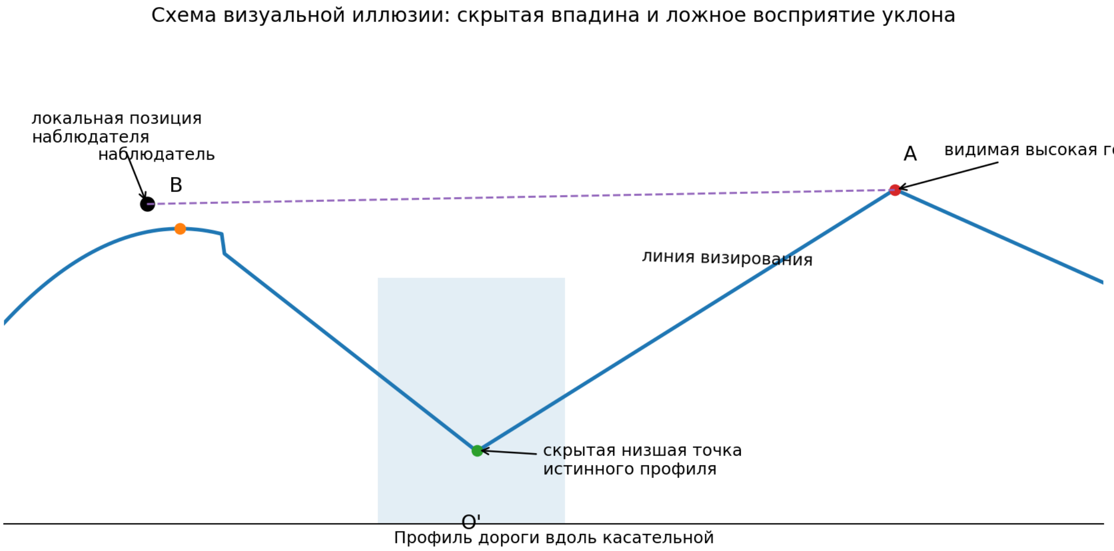

Figure 2: Visual-illusion scheme. The observer sees the distant high point A, but does not see the hidden lower point near O'. As a result, the true descending direction of the profile may be perceived as an ascent.

Remark 3.1. The geometric reason for the illusion is therefore not a change in the direction of the field. The error arises because the observer compares only those fragments of the profile that are available to sight, without seeing the global low point on the relevant interval.

4 Physical Interpretation

Interpret the point O as the center of a planet and the function Φ as a scalar quantity that strictly decreases with the distance from the center. Then the circles S_r are equipotential lines, and the tangency point O' is the distinguished point at which the potential along the tangent is maximal.

The physically correct conclusion is as follows:

- the mathematical model of a central field does not predict spontaneous motion from a nearer point toward a farther summit along the tangent;
- the observed “uphill flow” or “rising road” belongs to the class of perspective or visibility illusions;
- to explain the illusion, one has to analyze the visible profile rather than the local visual impression near the observer.

5 Conclusion

Two separate facts have been established.

First, for every central field of the form

$$\Phi(X) = f(|OX|),$$

with f strictly decreasing, the restriction of the potential to a tangent line of an equipotential circle attains a strict maximum at the point of tangency. This gives an exact rule for comparing all points on the tangent: the farther a point is from O', the smaller the value of Φ.

Second, the seeming reversal of the downhill direction is explained as a visual illusion caused by a hidden lower point of the profile. The phenomenon therefore does not indicate any violation of gravitational law. It reflects the difference between the true geometry of the profile and the incomplete image available to the observer.