

KASVD Spectral Geometry and Seven-Dimensional Anticommutative Algebras

A Matrix Bridge for KLT-RBD Reper Artifacts

Ivan B. Kurpishev

Kaliningrad, 2026

Abstract

We construct a rigorous mathematical layer connecting a finite typed graph of a Reper artifact with a matrix $M \in M_3(\mathbb{R})$, its singular spectrum, and a stratified KASVD state space. In parallel, we study the seven-dimensional anticommutative algebra

$$A_M = \mathbb{R}^3 \oplus \mathbb{R}^3 \oplus \mathbb{R}h,$$

in which the same matrix M defines a mixed central bilinear block. We prove invariance of the normalized SVD passport, a rank-gap statement, a rank-birth criterion for rank-one updates, the exact rank-one edit distance, a formula for the dimension of the derivation algebra, and rigidity with respect to the Binary-Lie, Malcev, and Sagle varieties:

$$A_M \in \text{BinaryLie} \iff A_M \in \text{Malcev} \iff A_M \in \text{Sagle} \iff M = 0.$$

We also introduce Kurpishev's authorial variational functional on a spectrally safe region. The mathematically proved spectral-algebraic layer is explicitly separated from the prospective/blind infrastructure of KLT-RBD. No empirical predictive validation is claimed in this paper.

Keywords: KASVD; KLT-RBD; Reper; singular value decomposition; anticommutative algebra; Binary-Lie; Malcev; Sagle; rank-one perturbation; variational principle.

MSC 2020: 17A30, 15A18, 15A60, 65F35.

1. Introduction

In the project LOGIC OF KURPISHEV 2, a Reper object is written as $\text{Rep} = (R, I, U; D)$, while KLT-RBD is used as a machine-auditable memory of sources, sufficient grounds, transitions, blockers, and statuses. The present paper isolates a self-contained mathematical problem from that larger corpus: to construct a

small matrix carrier on which rank, spectral symmetry, perturbation stability, and elementary structural transitions can be treated rigorously.

The central object is a matrix $M \in M_3(\mathbb{R})$. It appears in two independent ways. First, it is an operator assembled from typed edges of a finite Artifact graph. Second, it is the mixed bilinear block of a seven-dimensional anticommutative algebra A_M . These two levels are not semantically identified. Their mathematical relation is a common SVD invariant.

The classical background consists of singular value decomposition, the Eckart-Young-Mirsky theorem, standard singular-value perturbation bounds, and the established definitions of Binary-Lie, Malcev, and Sagle algebras [1-6]. The authorial layer consists of the Artifact-to-KASVD construction, the KASVD state stratification and transition interpretation, the use of one matrix M as a common module for the Artifact operator and A_M , and the variational functional $\mathcal{J}_K$.

2. Artifact operator and KASVD

Definition 2.1. Artifact matrix

Let $G_A = (V_A, E_A, \tau, w)$ be a finite typed weighted graph. Fix orthonormal three-dimensional channel spaces

$$X = \text{span}\{R, I, U\}, \quad Y = \text{span}\{D, P, G\}.$$

Here P and G are analytical proof/evidence and gap/blocker channels; they are not declared to be additional components of the Reper itself. To each edge e assign $x_e \in X$, $y_e \in Y$, and a weight $w(e)$. Define

$$T_A = \sum_{e \in E_A} w(e) x_e \otimes y_e, \quad M_A = [T_A] = \sum_{e \in E_A} w(e) x_e y_e^T.$$

Definition 2.2. KASVD passport

Let $M \neq 0$ and let

$$\sigma_1(M) \geq \sigma_2(M) \geq \sigma_3(M) \geq 0$$

be its singular values. Set

$$\rho(M) = \|M\|_F, \quad s_2 = \frac{\sigma_2}{\sigma_1}, \quad s_3 = \frac{\sigma_3}{\sigma_1}.$$

We call

$$\text{SpecRep}(M) = (\rho; s_2, s_3)$$

the spectral passport, and (s_2, s_3) the KASVD point. It lies in the closed triangle

$$0 \leq s_3 \leq s_2 \leq 1.$$

Proposition 2.3. Shape invariance

Under

$$M \mapsto cAMB^T, \quad A, B \in O(3), \quad c \neq 0,$$

the KASVD point (s_2, s_3) is preserved, while ρ is multiplied by $|c|$.

Proof. Orthogonal factors preserve singular values and multiplication by c scales all singular values by $|c|$. Their ratios to σ_1 are therefore unchanged. $\square$

Theorem 2.4. Rank-gap theorem

If the active edges of the Artifact graph use at most two independent input directions in X , or at most two independent output directions in Y , then

$$\text{rank } M_A \leq 2, \quad \sigma_3(M_A) = 0.$$

Proof. In the first case the effective domain of T_A has dimension at most two; in the second case the image of T_A is contained in a subspace of dimension at most two. Hence the rank is at most two. $\square$

The converse is false: $\sigma_3 = 0$ may also arise from linear dependence or cancellation of weights. Semantic, structural, and spectral gaps must therefore be stored separately.

3. Rank-one dynamics

Let

$$M' = M + E, \quad E = \alpha xy^T,$$

so that $\text{rank } E \leq 1$.

Theorem 3.1. Rank-birth criterion

Assume $\text{rank } M = 2$, and let unit vectors ℓ and r span $\ker M^T$ and $\ker M$, respectively. Then

$$\text{rank}(M + \alpha xy^T) = 3$$

if and only if

$$\alpha \neq 0, \quad \ell^T x \neq 0, \quad y^T r \neq 0.$$

Proof. A rank-one update raises the rank of a rank-two matrix exactly when the new left direction lies outside $\text{Col } M$ and the new right direction lies outside $\text{Row } M$. In codimension one these conditions are equivalent to the two stated nonzero scalar products. $\square$

Theorem 3.2. Rank-change bound

For a rank-one update,

$$|\text{rank}(M + E) - \text{rank } M| \leq 1.$$

Thus one elementary edge update cannot carry a rank-one state directly to a rank-three state.

Theorem 3.3. Exact rank-one edit distance

The minimum number of rank-one summands required to pass from a fixed matrix M to a fixed matrix N is

$$d_1(M, N) = \text{rank}(N - M).$$

Proof. A sum of k rank-one matrices has rank at most k , giving the lower bound. Conversely, the SVD of $N - M$ writes it as a sum of exactly $r = \text{rank}(N - M)$ rank-one terms. $\square$

4. KASVD stratification and adjacency

For a nonzero 3×3 matrix we use seven strata

$$S_1, S_{2g}, S_{2e}, S_{3g}, S_{3,12}, S_{3,23}, S_{3i}.$$

The index gives the rank; g denotes pairwise distinct positive singular values; e denotes equality of the two positive singular values in rank two; and 12, 23, and i denote the corresponding multiplicities in rank three.

Theorem 4.1. Existential rank-one adjacency

For two KASVD strata there exists at least one pair of representatives $M \in S_\alpha, N \in S_\beta$ such that

$$\text{rank}(N - M) \leq 1$$

if and only if

$$|\text{rank } S_\alpha - \text{rank } S_\beta| \leq 1.$$

Proof. Necessity follows from the standard rank inequality. For ranks one and two, diagonal representatives $\text{diag}(a, 0, 0)$ and $\text{diag}(a, b, 0)$ suffice, including the case $a = b$. For transitions from rank two to rank three, diagonal representatives cover generic and double strata. The transition $S_{2g} \rightarrow S_{3i}$ is realized, for example, by $I - uv^T \rightarrow I$ with $u = e_1$, $v = e_1 + e_2$; the source has singular values $(\sqrt{2}, 1, 0)$. Within the rank-three block, most cases are obtained by changing one diagonal coordinate; the transition $S_{3i} \rightarrow S_{3g}$ is realized by $I + e_1(e_1 + e_2)^T$, which has three distinct singular values. Reverse arrows follow by replacing E with $-E$. $\square$

The theorem is existential. For specific matrices M and N , the exact criterion remains $d_1(M, N) = \text{rank}(N - M)$.

5. Spectral stability

For $M' = M + E$, the standard singular-value perturbation bound gives [3,4]

$$|\sigma_k(M') - \sigma_k(M)| \leq \|E\|_2.$$

In the generic full-rank stratum define

$$m_R = \sigma_3, \quad m_{12} = \frac{\sigma_1 - \sigma_2}{2}, \quad m_{23} = \frac{\sigma_2 - \sigma_3}{2},$$

and

$$r_{\text{safe}}(M) = \min\{m_R, m_{12}, m_{23}\}.$$

Corollary 5.1. Safe-radius guarantee

If

$$\|E\|_2 < r_{\text{safe}}(M),$$

then $M + E$ remains full rank and does not cross either surface $\sigma_1 = \sigma_2$ or $\sigma_2 = \sigma_3$.

For a sequence of updates $E_1, \dots, E_n$, absence of intermediate crossings is guaranteed, for example, by the stronger condition

$$\sum_{j=1}^n \|E_j\|_2 < r_{\text{safe}}(M_0),$$

because it controls every prefix.

6. The seven-dimensional anticommutative algebra

Let

$$A_M = V \oplus W \oplus L, \quad V \cong W \cong \mathbb{R}^3, \quad L = \mathbb{R}h,$$

and define the anticommutative product

$$[(v, w, t), (v', w', t')] = (v \times v', w \times w', v^T M w' - v'^T M w).$$

The vector h spans a central annihilator. For $M = 0$ one obtains the Lie algebra

$$A_0 \cong \mathfrak{so}(3) \oplus \mathfrak{so}(3) \oplus \mathbb{R}h.$$

For $M \neq 0$, the mixed block generally violates the Jacobi identity.

Proposition 6.1. Orientation-preserving SVD equivalence

If $A, B \in SO(3)$ and the central coordinate is rescaled by a nonzero factor, the mixed block transforms as

$$M \mapsto cAMB^T, \quad c \neq 0.$$

Hence (s_2, s_3) is an invariant of this equivalence.

The Artifact operator admits the larger $O(3) \times O(3)$ action, whereas the cross product is naturally equivariant under $SO(3)$. Thus the common object of the two levels is the SVD invariant, not an identity of the full transformation groups.

7. Derivations

Theorem 7.1. Stabilizer of the mixed block

Let $M \neq 0$. Every derivation of A_M induces on $V \oplus W$ a block $P \oplus Q$ with

$$P, Q \in \mathfrak{so}(3),$$

and

$$PM = MQ.$$

If the positive singular values of M have multiplicities m_α , and

$$k = \dim \ker M,$$

then

$$\dim \text{Der}(A_M) = \sum_{\alpha} \binom{m_{\alpha}}{2} + k(k-1).$$

Proof. The annihilator L is characteristic. On the quotient $A_M/L \cong \mathfrak{so}(3) \oplus \mathfrak{so}(3)$ the induced derivation is inner, hence is represented by $P, Q \in \mathfrak{so}(3)$. Compatibility with the central bilinear form has the form $cM = -PM + MQ$. Taking the Frobenius inner product with M gives $c\|M\|_F^2 = 0$, because $\text{tr}(M^T PM)$ and $\text{tr}(M^T MQ)$ vanish for skew-symmetric P, Q . Hence $c = 0$. After SVD, the equation $PM = MQ$ splits into blocks of equal positive singular values and the kernel. A positive block of multiplicity m_{α} contributes $\binom{m_{\alpha}}{2}$ parameters, while two independent skew blocks on a k -dimensional kernel contribute $2\binom{k}{2} = k(k-1)$. $\square$

8. Binary-Lie, Malcev, and Sagle rigidity

Let

$$J(x, y, z) = [[x, y], z] + [[y, z], x] + [[z, x], y]$$

be the Jacobian.

Theorem 8.1. Binary-Lie rigidity

For the family A_M ,

$$A_M \in \text{BinaryLie} \iff M = 0.$$

Proof. In anticommutative notation the Binary-Lie identity is equivalent to [1,2]

$$J(x, y, [x, y]) = 0.$$

For every $i, j \in \{1, 2, 3\}$ choose any $k \neq j$ and set

$$x = f_k, \quad y = e_i - f_j.$$

A direct calculation gives

$$J(x, y, [x, y]) = M_{ij}h.$$

Thus all nine entries M_{ij} vanish. Conversely, $M = 0$ yields the Lie algebra A_0 , and every Lie algebra is Binary-Lie. $\square$

Corollary 8.2. Malcev rigidity

$$A_M \in \text{Malcev} \iff M = 0.$$

Proof. Every Malcev algebra is Binary-Lie [1,2]. Apply Theorem 8.1; the converse follows from the Lie case. $\square$

Theorem 8.3. Sagle rigidity

Write the Sagle identity in the form [1]

$$[J(x, y, z), w] = J(w, z, [x, y]) + J(w, y, [z, x]) + J(w, x, [y, z]).$$

Then

$$A_M \in \text{Sagle} \iff M = 0.$$

Proof. Define the Sagle defect

$$\mathcal{S}(x, y, z, w) = [J(x, y, z), w] - J(w, z, [x, y]) - J(w, y, [z, x]) - J(w, x, [y, z]).$$

For the standard bases e_1, e_2, e_3 of V and f_1, f_2, f_3 of W , direct substitution gives:

coefficient	(x, y, z, w)	$\mathcal{S}(x, y, z, w)$
M_{11}	(e_1, e_2, f_1, e_2)	$M_{11}h$
M_{12}	(e_1, e_2, f_2, e_2)	$M_{12}h$
M_{13}	(e_1, e_2, f_3, e_2)	$M_{13}h$
M_{21}	(e_1, e_2, f_1, e_1)	$-M_{21}h$
M_{22}	(e_1, e_2, f_2, e_1)	$-M_{22}h$
M_{23}	(e_1, e_2, f_3, e_1)	$-M_{23}h$
M_{31}	(e_1, e_2, f_2, f_3)	$-M_{31}h$
M_{32}	(e_1, e_2, f_1, f_3)	$M_{32}h$
M_{33}	(e_1, e_2, f_1, f_2)	$-M_{33}h$

If A_M is Sagle, every defect is zero, so $M_{ij} = 0$ for all i, j . Conversely, $M = 0$ yields a Lie algebra, and Lie algebras satisfy the Sagle identity. $\square$

Thus, inside the family A_M , three broader nonassociative classes collapse to the unique Lie layer:

$$A_M \in \text{BinaryLie} \iff A_M \in \text{Malcev} \iff A_M \in \text{Sagle} \iff M = 0.$$

9. The common SVD-moduli bridge

Theorem 9.1. Artifact/KASVD - A_M bridge

Let the same nonzero matrix M be used

1. as the matrix of an Artifact operator $T_A : X \rightarrow Y$;
2. as the mixed central block of the anticommutative algebra A_M .

Then the normalized spectrum

$$(s_2, s_3) = \left(\frac{\sigma_2}{\sigma_1}, \frac{\sigma_3}{\sigma_1} \right)$$

is a common invariant of the orthogonal-scale geometry of the Artifact operator and of the orientation-preserving orthogonal-scale equivalence of the algebraic mixed block.

Proof. At the Artifact level the singular ratios are invariant under $O(3) \times O(3)$ and a common nonzero scale. At the algebraic level the continuous basis action contains $SO(3) \times SO(3)$ together with the same nonzero rescaling of the central coordinate. In both cases all singular values are changed only by a common factor; hence their ratios agree. $\square$

The theorem does not assert that a semantic Artifact is an algebra. It asserts only that the two constructions possess a rigorous common matrix invariant.

10. Weighted KASVD and the variational layer

For a fixed matrix M and a target stratum T , define the local cost

$$c_M(T) = \inf\{\|E\|_2 : \text{rank } E \leq 1, M + E \in T\}.$$

For a full-rank matrix the minimum cost of rank loss is

$$\boxed{c_M(R_2) = \sigma_3,}$$

attained by $E = -\sigma_3 u_3 v_3^T$. In the reverse direction, from rank two to rank three, the infimal cost is zero: a new independent mode can be created by an arbitrarily small nonzero perturbation whose left and right directions both leave the column and row spaces.

On the generic region

$$\Omega_{gen} = \{\sigma_1 > \sigma_2 > \sigma_3 > 0\}$$

define

$$V_K(M) = -\alpha_R \log m_R - \alpha_{12} \log m_{12} - \alpha_{23} \log m_{23}, \quad \alpha_i > 0,$$

and Kurpishev’s authorial mathematical variational functional

$$\mathcal{J}_K[M] = \int_{t_0}^{t_1} \left(\frac{1}{2} \|\dot{M}\|_F^2 + V_K(M) \right) dt.$$

For simple singular values, $\nabla \sigma_i = u_i v_i^T$, so inside a regular stratum the formal Euler-Lagrange equation is

$$\ddot{M} = \nabla_F V_K(M).$$

Proposition 10.1. A well-posed existence setting

The logarithmic barrier alone is not coercive on the unbounded matrix space. On the compact safe region

$$\Omega_\delta = \{M : \|M\|_F = 1, m_R, m_{12}, m_{23} \geq \delta\},$$

a finite-dimensional discrete fixed-endpoint functional attains a minimum whenever the admissible trajectory set is nonempty.

Proof. Ω_δ is compact, a finite product of copies of it is compact, the fixed-endpoint admissible set is closed, and the discrete functional is continuous. The Weierstrass theorem applies. $\square$

11. Prospective certificate and the boundary of empirical claims

For KLT-RBD a computational contour has been constructed:

Source $\rightarrow$ Intake $\rightarrow$ Evidence-D $\rightarrow G_t \rightarrow M_t \rightarrow$ SVD $\rightarrow$ Transition $\rightarrow$ Certificate $\vee$ ABSTAIN.

The append-only ledger, hash-bound materialization, blind split, and scoring protocol make the computation reproducible, but they are not evidence of predictive power. Synthetic tests are used to test executability of the model, not as a substitute for a real holdout. At the time of this paper, real prospective validation has not been performed; empirical predictive claims therefore remain outside the proved layer.

12. Novelty and limitations

The following are classical and are not claimed as authorial discoveries: SVD, rank inequalities, the Eckart-Young-Mirsky theorem, standard perturbation bounds, the established Lie/Malcev/Binary-Lie/Sagle theory, and compactness arguments.

The authorial mathematical construction of I. B. Kurpishev in this paper consists of:

1. the map from a finite Reper Artifact graph to a three-channel operator M_A and its KASVD passport;
2. KASVD stratification as a KLT-RBD state layer together with its operational rank-one interpretation;
3. the use of one matrix M as a common module for the Artifact operator and the family A_M ;
4. explicit rigidity theorems for A_M , including nine Binary-Lie and nine Sagle witnesses;
5. weighted KASVD and the functional $\mathcal{J}_K$;
6. the strict separation between a computed certificate and truth-status.

Open problems include the full isomorphism group of A_M including discrete components, the global geometry of KASVD moduli near repeated singular values, and independent real validation of the prospective prediction layer.

13. Conclusion

A single rigorous matrix layer has been obtained in which the same 3×3 matrix carries two different structures: a spectral state of a Reper Artifact operator and the mixed central block of a seven-dimensional anticommutative algebra. The SVD ratios (s_2, s_3) form a common invariant of these constructions. For the algebraic family, a rigidity result is proved: every nonzero M moves A_M simultaneously outside the Binary-Lie, Malcev, and Sagle classes. For the Artifact layer, exact rank-one distances and safe spectral thresholds are obtained. The variational layer provides a mathematical trajectory model, while its empirical interpretation remains a separate prospective-verification problem.

Appendix A. Nine Binary-Lie witnesses

For $i, j \in \{1, 2, 3\}$ choose any $k \neq j$ and set

$$x = f_k, \quad y = e_i - f_j.$$

Then

$$J(x, y, [x, y]) = M_{ij}h.$$

For instance one may fix $k(1) = 2$, $k(2) = 3$, $k(3) = 2$, yielding nine deterministic tests of all entries of M .

Appendix B. Nine Sagle witnesses

Let $\mathcal{S}(x, y, z, w)$ denote the Sagle defect with all terms moved to one side. The nine quadruples listed in Theorem 8.3 yield respectively

$$M_{11}, M_{12}, M_{13}, -M_{21}, -M_{22}, -M_{23}, -M_{31}, M_{32}, -M_{33}$$

in the central h coordinate. Thus the Sagle identity annihilates each coordinate of M separately.

References

1. Kaygorodov, I.; Zusmanovich, P. On anticommutative algebras for which $[R_a, R_b]$ is a derivation. *Journal of Geometry and Physics* **166** (2021), 104113. DOI: 10.1016/j.geomphys.2021.104113.
2. Sagle, A. A. On simple extended Lie algebras over fields of characteristic zero. *Pacific Journal of Mathematics* **15**(2) (1965), 621-648.
3. Stewart, G. W.; Sun, J.-G. *Matrix Perturbation Theory*. Academic Press, 1990.
4. Horn, R. A.; Johnson, C. R. *Topics in Matrix Analysis*. Cambridge University Press, 1991.
5. Eckart, C.; Young, G. The approximation of one matrix by another of lower rank. *Psychometrika* **1** (1936), 211-218.
6. Mirsky, L. Symmetric gauge functions and unitarily invariant norms. *Quarterly Journal of Mathematics* **11** (1960), 50-59.
7. Kurpishev, I. B. *Basic Project KLT-RBD, Volume V: typed graph, provenance, versioning and audit mathematics*. Project corpus, 2026.
8. Kurpishev, I. B. *R3-Y Theorematic Consolidation and R4-A/R4-B Rigidity Packages*. LOGIC OF KURPISHEV 2 project corpus, 2026.